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On almost sure stability conditions of linear switching stochastic differential systems

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ABSTRACT

In modeling practical systems, it can be efficient to apply Poisson process and Wiener process to represent the abrupt changes and the environmental noise, respectively. Therefore, we consider the systems affected by these random processes and investigate their joint effects on stability. In order to apply Lyapunov stability method, we formulate the action of the infinitesimal generator corresponding to such a system. Then, we derive the almost sure stability conditions by using some fundamental convergence theorem. To illustrate the theoretical results, we construct an example to show that it is possible to achieve stabilization by using random perturbations.

1. Introduction

Switching systems can be used to describe some practical systems that undergo abrupt changes. Mathematically, such a system is composed of several subsystems with a switching signal to orchestrate among them. Therefore, one of the fundamental problems in the research area is to characterize the mechanism triggering switching and study its effect on stability. One way to deal with switching signal is to assume it to evolve according to the law of continuous-time Markov chain. In fact, this pure assumption implies that the occurrence of switching is a Poisson event; see, e.g., [1]. From this perspective, roughly speaking, the stability relies on two aspects, namely, the embedded Poisson distribution and the embedded discrete-time Markov chain; see, e.g., [2–4]. Indeed, the embedded Poisson distribution represents the density of switching points, while the embedded Markov chain indicates the likelihood of a subsystem to be activated at a switching point. In this sense, focusing on the aspect how the varying rate of switching signal influences stability, we might simply suppose the evolution of switching signal over time to obey Poisson distribution; see, e.g., [5].

Motivated by the above observation, we shall consider the systems affected by two basic kinds of Lévy processes, namely, Poisson processes and Wiener Processes. Thereby, we can in a unified stochastic framework account for the abrupt changes and the environment noise, which are commonly encountered in modeling practical systems. Recently, Chatterjee and Liberzon [6,7] proved that the solutions of such systems, if exist, have some nice statistic properties. In this paper, we make an attempt to investigate the joint effects of these random processes on the stability in almost sure sense. Within this context, as pointed out in [8], Wiener process can be used to stabilize a given unstable system or to make a system more stable even when it is already stable. Also, from [9,10] we learn that modulating random switching signal can influence the dynamical behavior and make it very different. In this paper, we observe that it is possible to achieve stabilization by letting Poisson and Wiener processes work together. Thus, we may gain a more comprehensive insight into the randomly perturbed stability problem.

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In parallel with the stochastic framework to account for switching, it is commonly used to classify switching signals in terms of average dwell-time. And it provides a conventional way to address the stability problem of switching systems by using multiple Lyapunov functions approach; see, e.g., [11] and the references therein. In this paper, we shall explain the connection between the deterministic and stochastic frameworks to deal with switching. In fact, we pose an analogy to the multiple Lyapunov functions approach. It requires us to formulate the infinitesimal generator of the systems under investigation, which plays a basic role for capturing the stochastic dynamics. Then, we derive the stability conditions by using some fundamental convergence theorem.

The remainder of the paper is organized as follows. In Section 2 we describe the system under consideration and formulate the problem. In Section 3, we present the stability conditions and interpret the construction of the switching signal that obeys Poisson distribution. In Section 4, a simulation example is worked out to illustrate the results. Finally, the paper is briefly summarized in Section 5.

2. Problem description

Consider the linear switching stochastic system described as follows

$$\mathrm{d}x(t) = A_{\sigma(t)}x(t)\mathrm{d}t + G_{\sigma(t)}x(t)\mathrm{d}w(t), \quad t \geq t_0 = 0, \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is state vector and $w(t)$ stands for the normalized one-dimensional Wiener Process, which is defined on the filtered complete probability space $\{\Omega, \mathcal{F}, \mathcal{F}_t, \mathcal{P}\}$. We suppose $\{\mathcal{F}_t\}_{t\geq 0}$ to satisfy the usual conditions, namely, each $\mathcal{F}_t$ is $\mathcal{P}$-complete and $\mathcal{F}_t = \cap_{s>t} \mathcal{F}_s$ for every t. Besides, write $\mathcal{E}$ for the mathematical expectation operator. System (1) is generated by the switching signal $\sigma(t)$ that orchestrates among the following subsystems

$$\mathrm{d}x(t) = A_i x(t)\mathrm{d}t + G_i x(t)\mathrm{d}w(t), \quad i \in \{1, \ldots, N\}.$$

According to the evolution of switching signal over time, it can be expanded into the following sequential form

$$\{(\sigma(t_0), t_0), (\sigma(t_1), t_1), \ldots, (\sigma(t_k), t_k), \ldots\}. \tag{2}$$

It means that the $\sigma(t_k)$th subsystem is activated during the interval $[t_k, t_{k+1})$. We now suppose the switching signal in (2) to obey Poisson distribution and to be independent of $w(t)$. That is, for any $\Delta > 0$ we have

$$\mathcal{P}\{t_{k+1} - t_k \geq \Delta\} = e^{-\lambda\Delta}, \quad k = 0, 1, 2, \ldots, \tag{3}$$

where $\lambda > 0$ is referred to as Poisson exponent. Therefore, the switching points $t_0 < t_1 < \cdots < t_k < \cdots$ turn out to constitute a sequence of stopping times tending to infinity. Equivalently, it reads that

$$\mathcal{P}\{\sigma(t + \Delta) \neq \sigma(t)\} = 1 - e^{-\lambda\Delta}. \tag{4}$$

As is well known, the Poisson exponent λ defines the mean value of the number of the switching points distributed within the time interval of unit length. Then it allows us to make a sense of the varying rate of switching signal. Accordingly, we categorize switching signals by means of λ. Namely, we write $\mathcal{S}_\lambda$ for the set of all the switching signals obeying Poisson distribution exactly with the exponent λ. For given switching signal σ, we denote by $x(t; x_0, \sigma)$ the corresponding motion of system (1) at time t starting from x_0 at initial time t_0.

Definition 1. System (1) is said to be almost surely exponentially stable, if

$$\mathcal{P}\left\{\limsup_{t\to\infty} \frac{\ln|x(t; x_0, \sigma)|}{t} < 0\right\} = 1$$

for all $x_0 \in \mathbf{R}^n$.

3. Main results

We aim at establishing the conditions that guarantee system (1) to be almost surely exponentially stable with respect to certain $\mathcal{S}_\lambda$. The starting point is to formulate the action of the infinitesimal generator for it corresponds to the one-parameter semigroup of operators, which in turn determine the state-transition. In particular, the infinitesimal generator plays a fundamental role for addressing the stochastic stability problem due to the fact that, in general, we are not be able to express the state-transition in an analytical closed form (cf. [12]). For the sake of generality, we shall first consider the switching stochastic differential system as follows

$$\mathrm{d}x(t) = f_{\sigma(t)}(x(t))\mathrm{d}t + g_{\sigma(t)}(x(t))\mathrm{d}w(t), \tag{5}$$

where the switching signal obeys the Poisson distribution as in (4). We suppose the drift coefficients $f_i(x)$ and the diffusion coefficients $g_i(x)$ to satisfy certain conditions so that we can guarantee the solution of (5) to exist on $[0, \infty)$. To each subsystem $\mathrm{d}x(t) = f_i(x(t))\mathrm{d}t + g_i(x(t))\mathrm{d}w(t)$, assign a positive-definite function $V_i(x)$, which has the partial derivatives