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Output tracking of switched Boolean networks under open-loop/closed-loop switching signals[☆]

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ABSTRACT

This paper addresses the output tracking problem of switched Boolean networks (SBNs) via the semi-tensor product method, and presents a number of new results. Firstly, the concept of switching-output-reachability is proposed for SBNs, based on which, a necessary and sufficient condition is presented for the output tracking of SBNs under arbitrary open-loop switching signal. Secondly, a constructive procedure is proposed for the design of closed-loop switching signals for SBNs to track a constant reference signal. The study of an illustrative example shows that the obtained new results are very effective.

1. Introduction

Boolean networks have attracted a great attention from biologists, physicists and systems scientists in the last half of a century [1–3]. Recently, a semi-tensor product method [4,5] has been established for the analysis and control of Boolean networks. The main contribution of this method is that one can convert the dynamics of a Boolean network into a linear discrete-time system, and then one can study Boolean networks by using the classical control theory. Using this novel method, many scientists have systematically investigated Boolean networks, and many excellent works have been obtained on the control of Boolean networks, which include controllability and observability [6–12], stability and stabilization [13,14], optimal control [15–17], and other control problems [18–26].

As was shown in [27–29], the dynamics of Boolean networks in practice is often governed by different switching models. For example, a Boolean control network can be regarded as a switched system by encoding the control inputs as a switching signal [8] (see Section 6). Moreover, an asynchronous Boolean network can be converted to a switched one by combining all the Boolean functions [30]. Thus, it is necessary for us to investigate switched Boolean networks (SBNs). In the last five years, some interesting results have been obtained for the controllability and stability of SBNs by using the semi-tensor product method. The controllability of SBNs was studied in [29], and a kind of switching-input-state incidence matrix was proposed for the controllability analysis. The output controllability and optimal output control of state-dependent SBNs were considered in [27], and some necessary and sufficient conditions were presented. The stability of SBNs under arbitrary switching signal was investigated in [28,30], and some necessary and sufficient conditions were established.

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It is noted that a basic issue in the control theory is to make the measured outputs of a plant track a desirable reference signal, which is called the output tracking problem. This problem is also very important for genetic regulatory networks. In many practical genetic regulatory systems, the state variables cannot be obtained directly due to the limitation of measurement conditions and the impact of immeasurable variables. In this case, one can use the measured outputs to track a desirable reference signal which corresponds to some desirable gene states. For example, in order to manipulate the large scale behavior of the lactose regulation system of the Escherichia coli bacteria, Julius et al. [31] proposed a novel feedback control architecture to make the fraction of induced cells in the population (the output of the system) attain a desired level (a given reference trajectory). As a suitable model of genetic regulatory networks, the output tracking problem of Boolean networks was studied in [15,22], respectively, and some necessary and sufficient conditions were presented. However, to our best knowledge, there are no results available on the output tracking of SBNs.

In this paper, using the semi-tensor product method, we investigate the output tracking of SBNs under open-loop/closed-loop switching signals, and present a number of new results. The main contributions of this paper are as follows. (i) The output tracking of SBNs under arbitrary open-loop switching signal is firstly studied in this work, and a necessary and sufficient condition is presented for this problem based on the switching-output-reachability. One can easily verify the condition with the help of MATLAB toolbox. (ii) Based on the construction of a series of matrices, a constructive procedure is proposed to design closed-loop switching signals for SBNs to track a constant reference signal. The procedure is computationally tractable.

The rest of this paper is organized as follows. Section 2 recalls some necessary preliminaries on the semi-tensor product of matrices. Section 3 formulates the output tracking problem studied in this paper. Section 4 investigates the output tracking of SBNs under arbitrary open-loop switching signal, while Section 5 studies the output tracking of SBNs under closed-loop switching signals. An illustrative example is given to support our new results in Section 6, which is followed by a brief conclusion in Section 7.

Notation: $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{Z}_+$ denote the sets of real numbers, natural numbers and positive integers, respectively. $\mathcal{D} := \{1, 0\}$. $\mathbf{1}_n := [\underbrace{1\ 1\ \cdots\ 1}_{n}]$. $\Delta_n := \{\delta_n^k : k = 1, \ldots, n\}$, where δ_n^k denotes the kth column of the identity matrix I_n. For compactness, $\Delta := \Delta_2$. An $n \times t$ matrix M is called a logical matrix, if $M = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_t}]$, which is briefly expressed as $M = \delta_n[i_1\ i_2\ \cdots\ i_t]$. Denote the set of $n \times t$ logical matrices by $\mathcal{L}_{n\times t}$. $Col_i(A)$ denotes the ith column of the matrix A, and $Row_i(A)$ stands for the ith row of the matrix A. $Blk_i(A)$ denotes the ith $n \times p$ block of an $n \times mp$ matrix A.

2. Preliminaries

In this section, we recall some necessary preliminaries on the semi-tensor product of matrices.

Definition 2.1 ([4]). The semi-tensor product of two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is

$$A \ltimes B = \left(A \otimes I_{\frac{\alpha}{n}}\right)\left(B \otimes I_{\frac{\alpha}{p}}\right), \tag{2.1}$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, I_n denotes the $n \times n$ identity matrix, and $\otimes$ is the Kronecker product.

When $n = p$, the semi-tensor product of A and B becomes the conventional matrix product. Thus, it is a generalization of the conventional matrix product. We omit the symbol "$\ltimes$" if no confusion arises in the following. The semi-tensor product of matrices has the following properties.

Proposition 2.2 ([4]).

(i) *Let $X \in \mathbb{R}^{t\times 1}$ be a column vector and $A \in \mathbb{R}^{m\times n}$. Then*

$$X \ltimes A = (I_t \otimes A) \ltimes X. \tag{2.2}$$

(ii) *Let $X \in \mathbb{R}^{m\times 1}$ and $Y \in \mathbb{R}^{n\times 1}$ be two column vectors. Then*

$$Y \ltimes X = W_{[m,n]} \ltimes X \ltimes Y, \tag{2.3}$$

where

$$\begin{aligned} W_{[m,n]} = \delta_{mn}[&1\ \ m+1\ \ \cdots\ \ (n-1)m+1 \\ &2\ \ m+2\ \ \cdots\ \ (n-1)m+2 \\ &\qquad\quad \cdots \\ &m\ \ m+m\ \cdots\ (n-1)m+m] \in \mathcal{L}_{mn\times mn} \end{aligned}$$

is the so-called swap matrix.

Identify $1 \sim \delta_2^1$ and $0 \sim \delta_2^2$, then $\Delta \sim \mathcal{D}$, where "$\sim$" denotes two different forms of the same object. In most places of this work, we use δ_2^1 and δ_2^2 to express logical variables and call them the vector form of logical variables. The following lemma is fundamental for the matrix expression of logical functions.