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Generalized average dwell time approach to stability and input-to-state stability of hybrid impulsive stochastic differential systems

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ABSTRACT

This paper is concerned with the stability properties of a class of impulsive stochastic differential systems with Markovian switching. Employing the generalized average dwell time (gADT) approach, some criteria on the global asymptotic stability in probability and the stochastic input-to-state stability of the systems under consideration are established. Two numerical examples are given to illustrate the effectiveness of the theoretical results, as well as the effects of the impulses and the Markovian switching on the systems stability.

1. Introduction

Input-to-state stability (ISS), which can be traced to late 1980s [1], has sparked interests of many researchers due to its important role in the analysis and control of nonlinear systems [2–5]. For a system with ISS property, the norm of its solution should be upper-bounded by a vanishing transient term depending on the initial state, plus a term which is somewhat proportional to the norm of the external input [6]. Therefore, ISS implies not only that the unforced system (i.e., zero input system) is asymptotically stable in the Lyapunov sense but also that its behavior remains bounded when its exogenous input is bounded [7,8]. In recent years, various invariants of ISS have been investigated quite intensively including integral ISS [9–11], local ISS [12,13], regional ISS [14], exponential-weighted ISS [15], and strong iISS [6,16].

As is well known, both impulsive effects [17] and stochastic disturbances [18] are widely observed in modeling real world processes. Sometimes, these two phenomena are found existing in one process simultaneously. So far, many research studies have been carried out on the ISS of impulsive differential systems [19–22], stochastic differential systems [23–25], and impulsive stochastic differential systems (ISDSs) [26,27]. In particular, the pioneer research on the ISS of impulsive differential systems was conducted in [19], which proved that an impulsive differential system possessing an exponential ISS-Lyapunov function is uniformly ISS over impulse sequences satisfying the following *average dwell-time* (ADT) condition

$$-dN_\zeta(t,s)-(c-\lambda)(t-s)\leqslant\mu,\quad \forall\, t>s\geqslant t_0, \tag{1}$$

where $N_\zeta(t,s)$ denotes the number of impulse times in the semi-open interval $(s,t]$, and $c, d\in\mathbb{R}$ are the rate coefficients of the exponential ISS-Lyapunov function, μ, λ are a pair of arbitrary positive constants. Subsequently, under the ADT condition

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proposed in [19], Ref. [20] further investigated the ISS of impulsive differential systems with time delays by employing exponential ISS Lyapunov–Razumikhin functions and exponential ISS Lyapunov–Krasovskii functionals. To weaken the ADT condition, [21] introduced the following so-called *generalized ADT* (gADT) condition

$$-dN_{\zeta}(t,s) - c(t-s) \leqslant \ln h(t-s), \quad \forall t > s \geqslant t_0, \tag{2}$$

where h is an arbitrary function from $\mathbb{R}_+$ to $(0, \infty)$, for which there exists a function $\tilde{h} \in \mathcal{L}$ (i.e., $\tilde{h} : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and strictly decreasing with $\lim_{t\to\infty} \tilde{h}(t) = 0$) such that $h(t) \leqslant \tilde{h}(t)$, $\forall t \in \mathbb{R}_+$. Obviously, the gADT condition (2) includes the ADT condition (1) as a special case with $h(t) = e^{\mu-\lambda t}$. Compared with [19], [21] proved that an impulsive differential system could reserve the uniform ISS even if the ADT condition is replaced by the gADT condition. In addition, Ref. [22] considered the ISS of impulsive delay systems with switching and established a Lyapunov–Krasovskii-type ISS theorem, which shows that the ISS property of an impulsive switched delay system can be retained if the activation time for the non-ISS subsystems satisfies an upper bound and the dwell time for the ISS subsystems satisfies a lower bound condition. As for stochastic systems, [23] studied the practical stochastic ISS (SISS) of cascaded stochastic systems by applying stochastic analysis techniques. And [24] investigated the SISS and the global asymptotic stability in probability (GASiP) for switched stochastic systems based on a comparison principle. While in [25], a Razumikhin-type theorem on the pth moment ISS was developed for stochastic retarded systems with Markovian switching. In recent years, based on the above ISS results for impulsive differential systems or stochastic differential systems, some efforts have been spent on the ISS theory of ISDSs. For example, on the foundation of [24], Ref. [26] investigated the finite-time GASiP and finite-time SISS of switched stochastic differential systems with or without impulses. And [27] established a set of Lyapunov-based sufficient conditions for the SISS and the GASiP of ISDSs by utilizing the ADT condition (1).

However, except for [25], the works mentioned above did not pay attention to the Markov switching in the system models. Markov switching is appropriate to describe abrupt variation in system structures or parameters, which may be caused by sudden environment changes, subsystem switching, executor faults and failures that occurred in components or interconnections, etc. Markov switching systems have been widely investigated by many researchers, see, [28–32] for example. Especially, the theory of ISDSs with Markov switching, also known as hybrid ISDSs (HISDSs), has developed in a variety of directions in the past decade [33–36]. But unfortunately, to the best of our knowledge, little research on the ISS properties of this class of systems has been reported. In the study of the ISS theory of HISDSs, the following two questions need to be answered: (1) Is there a restriction between the transition rate of the Markov switching and the impulsive effects? (2) How would the ISS property of an HISDS be influenced by those of its subsystems? The answers to the two questions may well demonstrate the inherent features of the system brought by the Markov switching and the impulses.

Motivated by the above discussion, the purpose of this paper is to investigate the SISS property of HISDSs. It is known from the definitions that an SISS system must be GASiP when the external input is zero. So the study of the GASiP of the zero-input system would be very helpful to the SISS analysis of the corresponding disturbed system. Therefore, in the present paper, we shall first establish some sufficient conditions for the GASiP of the zero-input systems as a foundation for the SISS analysis of the disturbed systems. We point out that this paper can be viewed as an extension of the authors' work [27], in which the SISS of ISDSs without Markov switching was investigated. Compared with [27], the main contributions of this paper are mainly threefold: (1) More complicated techniques such as generalized Itô's formula and Lyapunov functions depending on the Markov chain are involved. (2) Mutual restraints between the impulses and the transition rate of the Markov chain are derived. (3) The gADT condition used in this paper is less conservative than the ADT condition in [27] and enlarges the admissible impulse sequences set.

This paper proceeds as follows. Section 2 recalls some notations and preliminaries. Section 3 develops the GASiP and the SISS criteria for HISDSs, followed in Section 4 by some brief discussions on the gADT condition and the effects of the Markov switching. Two numerical examples are provided in Section 5 and conclusions are drawn in Section 6.

2. Preliminaries

Throughout this paper, unless otherwise specified, $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}_+ = [0, +\infty)$, $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{R}^n$ denotes the n-dimensional real space equipped with the Euclidean norm $|\cdot|$, and $\mathcal{L}_{\infty}^{m}$ denotes the set of all locally essentially bounded function $u : \mathbb{R}_+ \to \mathbb{R}^m$ with norm $\|u\|_{\infty} = \operatorname{ess\,sup}_{t\geqslant t_0} |u(t)|$. A^{T} stands for the transpose of a vector or matrix A. We use the standard definitions for $\mathcal{K}$, $\mathcal{K}_{\infty}$ and $\mathcal{KL}$ (please refer to [37]). Denote by $\alpha \in v\mathcal{K}_{\infty}$ if $\alpha \in \mathcal{K}_{\infty}$ and α is convex. $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{L}$, if it is continuous and strictly decreasing with $\lim_{t\to\infty} \gamma(t) = 0$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions, and $\mathbb{E}[\cdot]$ be the expectation operator with respect to the probability measure $\mathbb{P}$. Let $B(t) = (B_1(t), \ldots, B_d(t))^{\mathrm{T}}$ be a d-dimensional Brownian motion defined on the space. Let $r(t)$, $t \geqslant t_0$ be a right-continuous Markov chain on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ taking values in a finite space $\mathbb{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geqslant 0$ is the transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$. We assume that the Markov chain $r(\cdot)$ is $\mathcal{F}_t$-adapted but independent of the Brownian motion $B(\cdot)$.