Nonlinear Analysis: Hybrid Systems 22 (2016) 228–248

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

State and dynamic output feedback control of switched linear systems via a mixed time and state-dependent switching law

Ding Zhai, An-Yang Lu, Jing-Hao Li, Qing-Ling Zhang *

College of Sciences, Northeastern University, Shenyang 110819, PR China
State Key Laboratory of Synthetical Automation for Process Industries, PR China

ARTICLE INFO

Article history:
Received 5 August 2015
Accepted 29 April 2016

Keywords:
Switched systems
Robust control
Average dwell time
Lyapunov–Metzler inequality
Discretized Lyapunov function

ABSTRACT

This paper investigates the problem of robust control for switched linear systems with dwell-time constraint. First, a mixed time-dependent and state-dependent switching law is proposed. The stability analysis and weighted L_2 gain analysis are proposed based on the Lyapunov–Metzler inequality method and average dwell-time technique. Then by using discretized Lyapunov function technique, the state feedback controllers and dynamic output feedback controllers are designed to stabilize the switched linear systems and satisfy a weighted L_2 gain performance under the proposed dwell-time switching law. The method proposed in this paper can reduce the numbers of the switches. Finally the effectiveness of proposed method is illustrated through three simulation examples.

1. Introduction

Switched systems, as one of the most important hybrid systems, consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems [1]. In recent years, the study of switched systems has attracted the attention of the scientific community [2–6]. Various methods have been proposed for studying the switched systems, such as the common Lyapunov function approach [7], multiple Lyapunov functions approach [8,9], state-dependent switching technique [10,11], average dwell-time(ADT) technique [12–14]. Besides, many problems for the switched systems have been studied, such as stability analysis [15,16], dynamic output feedback control [17,18], H_∞ control [19,20], fault detection [21,22] and so on.

The state-dependent switching problem for switched systems was first discussed in [23] and the state-dependent switching law was first considered in [24]. What is more, the stability condition expressed in terms of the Lyapunov–Metzler inequalities was proposed in [24]. And the most important merit of this technique is that this condition does not require each subsystem to be stable and contains the quadratic stability condition as a special case. Based on this approach, state-feedback and dynamic output-feedback control designs are developed in [25,17] respectively. Moreover, for the continuous-time switched systems, L_2 gain analysis and state feedback control synthesis can be found in [19,26,8] and in the case of dynamic output feedback design, it is important to cite [17]. It should be noted that compared to the time-dependent switching method, for example ADT technique which obeys an average dwell-time constraint, the state-dependent switching method does not guarantee any minimal dwell-time. This means that the switched systems may switch very fast under state-dependent switching laws. However, in practice, a minimal time period between consecutive switchings is always required. To overcome this problem, some techniques have been proposed in [27,28]. To make sure that the switched systems do

* Corresponding author at: College of Sciences, Northeastern University, Shenyang 110819, PR China.
E-mail address: qlzhang@mail.neu.edu.cn (Q.-L. Zhang).

http://dx.doi.org/10.1016/j.nahs.2016.04.007
1751-570X/

not switch too fast, in [27], a maximum average dwell time was introduced and a state-dependent switching strategy with dwell-time was proposed, and in [28], a relaxed min-switching logic and modified the Lyapunov–Metzler inequalities were proposed. However, this method will degrade the performance of the switched systems and implementing the state-dependent switching rule requires the state information. It is obvious that the requirement of the state information for output feedback control is unreasonable.

Nowadays, a discretized Lyapunov function(DLF) technique has been widely used [29–31]. The basic idea of the DLF technique is dividing the domain of definition of matrix function into finite discrete points or smaller regions, thus reducing the choice of time-scheduled Lyapunov function into choosing a finite number of parameters. By citing [32] which considers the state feedback control problem, if the state-dependent switching law obeys a dwell-time constraint, the DLF technique can be used to obtain a smaller L_2 gain. Moreover, the DLF technique was also used for some other problems of switched systems, for example [33].

Therefore, in this paper, to solve the problem that for some practical cases where a minimal time period between consecutive switchings is required, a new dwell-time switching method is proposed based on state-dependent switching approach, ADT technique and DLF technique. Similar to [27,34], state-dependent switching approach and ADT technique are used for switched systems simultaneously. One of the contributions of this paper is that there is no requirement of the state information for output feedback control which is better than [27]. Besides, for state feedback control, the proposed method may not reduce the performance with the appropriate design parameters. What is more, combined with the method in [27], a longer dwell-time will be permitted. The third contribution is that the proposed method reduces the number of switches. Finally, two numerical examples and a spherical inverted pendulum example are given to show the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, the system description, the definition of DLF and the preliminary are presented. The main results are expressed in Section 3. The first part of Section 3 is stability and weighted L_2 gain analysis, the second part is the state feedback controller design and the third part is the dynamic output feedback controller design. In Section 4, three examples are given.

Notation. Standard notations are used in this paper. For a matrix P, P^T denotes its transpose and $He(P) \triangleq P + P^T$. $P > 0$ and $P < 0$ denote positive definiteness and negative definiteness, respectively. $\mathcal{M}$ denotes the set of all Metzler matrices, composed by square matrices $\Pi \in \mathbb{R}^{N\times N}$ of fixed dimensions with elements π_{ij}, such that $\pi_{ij} \geq 0$ for all $j \neq i \in \{1, 2, \ldots, N\}$ and $\sum_{i=1}^{N} \pi_{ij} = 0$ for all $j \in \{1, 2, \ldots, N\}$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space.

2. Preliminaries

2.1. System description

Consider the following continuous-time switched linear systems

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + B_{d\sigma(t)}d(t) \\ y(t) &= C_{\sigma(t)}x(t) + D_{d\sigma(t)}d(t) \\ z(t) &= E_{\sigma(t)}x(t) + F_{\sigma(t)}u(t) + F_{d\sigma(t)}d(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^q$ is the measured output, $z(t) \in \mathbb{R}^m$ is the controlled output, $d(t) \in \mathbb{R}^d$ is the disturbance input. Piecewise constant function $\sigma(t)$ which takes values in the finite set $\mathbb{N} = \{1, 2, \ldots, r\}, r > 1$, is the switching signal and $\sigma(t) = i$ means that the ith subsystem is activated. The switching instants are expressed by a sequence $\{t_0, t_1, \ldots, t_k, \ldots\}$ where t_0 denotes the initial time and t_k denotes the kth switching instant and the i_kth subsystem is activated when $t \in [t_k,\ t_{k+1})$. $A_i, B_i, B_{di}, C_i, D_{di}, E_i, F_{di}$ are constant matrices of the appropriate dimensions.

Assumption 1. The switched systems are assumed to have a minimum dwell time T.

Definition 1 (*[35]*). For any $t_2 > t_1 \geq 0$, $N_1(t_1, t_2)$ denotes the number of discontinuities of the Lyapunov function over (t_1, t_2). If $N_1(t_1, t_2) \leq N_{10} + (t_2 - t_1)/T_1$ holds for $T_1 > 0$, $N_{10} \geq 0$. Then, T_1 is called the average dwell time. Without loss of generality, in this paper we choose $N_{10} = 0$ as in [36].

Definition 2 (*[27]*). $N_2(t_1, t_2) < \infty$ denote the number of switchings of a switching signal $\sigma(t)$ on the interval (t_1, t_2), and $T(t_1, t_2)$ denote the total dwell time on the interval (t_1, t_2). We say that $\sigma(t)$ has a maximum average dwell time (MADT) T_2 if $T(t_1, t_2) \leq N_2(t_1, t_2)T_2$, $\forall t_2 \geq t_1 \geq 0$.

Remark 1. If the switched systems are assumed to have a minimum dwell time T, appropriate T_1 and T_2 should be chosen such that $T = T_1 + T_2$.