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Delay-interval-dependent passivity analysis of stochastic neural networks with Markovian jumping parameters and time delay in the leakage term[☆]

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ABSTRACT

In this paper, the problem of passivity analysis is investigated for a class of stochastic neural networks with Markovian jumping parameters and time delay in the leakage term. The discrete delay is assumed to be time-varying and belongs to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. By constructing appropriate Lyapunov–Krasovskii functionals, and employing Newton–Leibniz formulation and the free weighting matrix method, several delay-dependent criteria for checking the passivity of the addressed neural networks are established in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. Two examples are given to show the effectiveness and less conservatism of the proposed criteria.

1. Introduction

In the past few decades, neural networks have been widely applied to various fields such as load frequency control in power systems, finance, fixed-point computations, mechanics of structures and materials, smart antenna arrays, and other scientific areas [1–3]. Therefore, neural networks play important roles in many practical systems. It has been recognized that time-delays are inherent features of many physical processes and often encountered in engineering systems, so their presence must be considered in realistic stability analysis.

It is well-known that time delays are always unavoidably encountered in the implementation of neural networks due to the finite switching speed of neurons and amplifiers. So the issue of stability analysis of neural networks with time delays attracts many researchers and a large number of stability results have been reported in the literature [4–8]. The obtained results can be classified into two types: delay-dependent criteria [6–9] and delay-independent criteria [10]. Generally speaking, delay-dependent stability criteria are usually less conservative than delay-independent ones especially when

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the size of the delay is small. And, pursuing the delay-dependent stability criteria is of much theoretical and practical value. On the other hand, stochastic systems have received much attention since stochastic modelling has come to play an important role in many branches of science and engineering applications. Since many practical systems can be modelled as stochastic differential equations with time delays, increasing efforts have been devoted to the study of stochastic time-delay systems [11–16].

The concept of passivity has played an important role in the analysis of the stability of dynamical systems, nonlinear control, and other research areas. The essence of the passivity theory is that the passive properties of a system can keep the system internal stability. So it gives a way to study nonlinear systems only by means of the general characteristics of the input and output dynamics. Recently, the passivity analysis problem for various neural networks was widely investigated in the literature [17–22]. In [18], Chen et al. presented both delay-independent and delay-dependent passivity conditions for stochastic neural network in the sense of mean square. In [19], authors studied the passivity analysis of neutral-type neural network with discrete and distributed time-varying delays with less decision variables. In [20], the passivity problem of memristor-based neural networks with time-varying delay is investigated. In [21], Zeng et al. investigated the delay-dependent passivity criteria for neural networks with both discrete and distributed delays using augmented Lyapunov functional and reciprocally convex approach. In [22], Raja et al. presented the passivity analysis for uncertain discrete time stochastic BAM neural networks with time-varying delays by using the new Lyapunov–Krasovskii functional and free-weighting matrix approach.

As pointed out by Gopalsamy in [23], the time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Like the traditional time delays, the leakage delays also have a great impact on the dynamics of neural networks and many works have appeared in the literature, see [24–28]. In [24], Gopalsamy initially investigated the BAM neural networks with leakage delays and obtained some sufficient conditions to guarantee the existence and global stability of a unique equilibrium point by employing M-matrices theory. Then based on this work, Peng [25] further studied the existence and global stability of periodic solutions for BAM neural networks with leakage delays by using the continuation theorem in coincidence degree theory and the Lyapunov functional. In [26,27], the authors studied the equilibrium point of two classes fuzzy neural networks with delays in leakage terms. By using the topological degree theory, delay-dependent stability conditions of neural networks of neutral type with time delays in the leakage term were proposed in [28]. Therefore, it is valuable to investigate the passivity analysis of neural networks with time delays in the leakage term. However, there has been very little existing works appeared for the passivity analysis of neural networks with leakage delay in the literature [29–31]. The passivity properties of uncertain neural networks with leakage delays and time-varying delays have been studied in [29]. In [30], the authors investigated the problem for passivity analysis of neutral type neural networks with Markovian jumping parameters and time delay in the leakage term. Unfortunately, the authors in these works neglected the effects of stochastic disturbances, which have also an important effect on the passivity analysis of neural networks. But in [31], Zhao et al. presented the passivity problem for stochastic neural networks with time-varying delays and leakage delays by using the Lyapunov functional and free-weighting matrix method.

Markovian jump systems are a special class of hybrid systems, which can be described by a set of linear systems with the transitions between models determined by a Markovian chain in a finite mode set. This kind of systems has applications in economic systems, modelling production systems and other practical systems. Recently, there have appeared a few works on the passivity analysis of neural networks with Markovian jumping parameters in the literature [32,33].

On the other hand, interval time-varying delays $0 \leq \tau_1 \leq \tau(t) \leq \tau_2$ are used to indicate that the propagated speed of signals is finite and uncertain in systems, such as network control systems. Under the interval time-varying delay condition, the delay-dependent passivity criteria for neural network was proposed in [30,32]. To the best of our knowledge, so far, no result on the delay-interval-dependent passivity analysis for Markovian jumping stochastic neural network with leakage delay is available in the existing literature. This motivates our present study.

This paper aims to develop delay-dependent passivity analysis of a class of Markovian jumping stochastic neural networks with time delays in the leakage term. By using the Lyapunov–Krasovskii functional technique, new delay-dependent passivity conditions are derived in terms of LMIs, which can be easily checked by MATLAB LMI toolbox. Moreover, numerical examples are provided to illustrate the effectiveness of the proposed criteria.

Notations. Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X \geq Y$ (respectively, $X > Y$), means that the matrix $X - Y$ is positive semi definite (respectively, positive definite). Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be the complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all $\mathcal{P}$- null sets and is right continuous). $\omega(t)$ be a scalar Brownian motion defined on the probability space. $\mathcal{E}[\cdot]$ is the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. The notation $*$ always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following stochastic neural networks with leakage delays and Markovian jumping parameters

$$dx(t) = \Big[-A(r(t))x(t-\delta) + B(r(t))g(x(t)) + C(r(t))g(x(t-\tau(t))) + u(t)\Big]\,dt$$
$$+\ \sigma(x(t), x(t-\tau(t)), t, r(t))\,d\omega(t),$$