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Hybrid chaos control of continuous unified chaotic systems using discrete rippling sliding mode control

Jun-Juh Yan [a,*], Chih-Yung Chen [a], Jason Sheng-Hong Tsai [b]

[a] Department of Computer and Communication, Shu-Te University, Kaohsiung 824, Taiwan, ROC

[b] Department of Electrical Engineering, National Cheng-Kung University, Tainan 701, Taiwan, ROC

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ABSTRACT

In this paper, a new systematic design procedure to stabilize continuous unified chaotic systems based on discrete sliding mode control (DSMC) is presented. In contrast to the previous works, the concept of rippling control is newly introduced such that the design of DSMC can be simplified and only a single controller is needed to realize chaos suppression. As expected, under the proposed DSMC law, the unified system can be stabilized in a manner of ripple effect, even when the external uncertainty is present. Last, two examples are included to illustrate the effectiveness of the proposed rippling DSMC developed in this paper.

1. Introduction

A chaotic system is a complex dynamic nonlinear system. Over the last two decades, chaos control has become an interesting topic since the pioneering research of Ott, Grebogi and Yorke [1]. As well known, the chaos phenomenon is an unusual response of complex nonlinear systems which arose from sensitive to initial conditions. Moreover, the chaos phenomenon also has broad Fourier transform spectra and its fractal property in phase plane [2,3]. However, the chaos phenomenon in some engineering system is highly unexpected for its applications. Therefore, various effective methods have been proposed to cope with the problems of control and stabilization for chaotic systems, such as sliding mode control [4–9], backstepping design technique [10,11], state feedback control [12,13], prediction-based control [14], optimal control [15,16], etc. On the other hand, due to the progress of computer and digital signal processing (DSP) technology, using them to implement the controller has become more and more popular and important. Therefore, research in discrete-time control has become intensified in recent years [17–19]. In a robust control system, sliding mode control is frequently adopted because it can offer many inherent advantages, such as fast response, good transient performance and insensitive to variation in plant parameters or external disturbances. Several design methods of both continuous and discrete sliding mode control have been found in the literature [4–9,17–19].

Motivated by the aforementioned, this study aims to present a control scheme to suppress chaos for unified chaotic systems (UCS) based on DSMC. The UCS is one of the paradigms of chaos since it exhibits a lot of nonlinear dynamics phenomena such as bifurcations and chaos. The UCS bridges the gap between the Lorenz system and the Chen system [20]. In this paper, hybrid discrete controlled system is considered, where a discrete type controller is proposed for a continuous UCS such that the designed DSMC can be easily implemented by digital devices. Furthermore, in contrast to the previous works [17–19], we introduce a new control concept so-called rippling control, that is, only a certain state is first controlled

* Corresponding author. Fax: +886 7 6154899.
E-mail address: jjyan@stu.edu.tw (J.-J. Yan).

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and when the specified state is stabilized, it will affect the other states like ripple and achieve the goal of control in a manner of ripple effect. This method can effectively reduce the control inputs but still achieve chaos suppression, which has remarkable significance in reducing the complexity for controller realization.

The rest of this paper is organized as follows: In Section 2, the analytic approach to discretize continuous chaotic systems is first introduced and the chaos control problem is formulated. In Section 3, based on the DSMC, a discrete switching surface design is firstly presented and the stability of the controlled system in the sliding mode is derived. Then a controller is proposed to achieve the hitting. The concept of rippling control is introduced in this section such that the controller can be simplified and only a single controller is enough to realize chaos suppression. In Section 4, we show an illustrative example. Finally, conclusions are presented in Section 5. Throughout this paper, it is noted that, $|w|$ represents the absolute value of w and $sign(s)$ is the sign function of s, if $s > 0$, $sign(s) = 1$; if $s = 0$, $sign(s) = 0$; if $s < 0$, $sign(s) = -1$.

2. System description and problem formulation

For completeness of the hybrid control system, in what follows the optimal discretizing method is first described. Consider a class of continuous chaotic systems described by

$$\dot{x}(t) = Ax + Bg(x(t), t) \tag{1}$$

where $x(t) \in R^n$ is the state vector, $g(x(t), t) \in R^r$ represents the nonlinear function vector. Matrices A and B are known constant matrices with appropriate dimensions. Then the discrete-time model of system (1) is given as [18]

$$x_d(k+1)T = Gx_d(kT) + Hg(x_d(kT)) \tag{2}$$

where T is the sampling time; $G = e^{AT}$; $H = [G - I_n]A^{-1}B$.

This study aims to propose a rippling DSMC to copy with the chaos suppression problem of chaotic systems. We consider the following UCS [20]:

$$\begin{aligned}
&\dot{x}_1(t) = (10 + 25\omega) \cdot [x_2(t) - x_1(t)] \\
&\dot{x}_2(t) = (28 - 35\omega)x_1(t) + (\omega - 1)x_2(t) - x_1(t)x_3(t) \\
&\dot{x}_3(t) = \left(-\frac{8}{3} - \frac{1}{3}\omega\right) x_3(t) + x_1(t)x_2(t) \\
&\begin{bmatrix} x_1(0) & x_2(0) & x_3(0) \end{bmatrix}^T = \begin{bmatrix} x_{10} & x_{20} & x_{30} \end{bmatrix}^T
\end{aligned} \tag{3}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T \in R^3$ is state vector, $\begin{bmatrix} x_{10} & x_{20} & x_{30} \end{bmatrix}^T$ is the initial value vector, and ω is the system parameter with $0 \le \omega < 1$. According to the value of parameter ω, the whole family of UCS (3) includes the generalized Lorenz system when $0 \le \omega < 0.8$, the generalized Chen system when $0.8 < \omega \le 1$ and the Lü system when $\omega = 0.8$. The dynamics of this unified system has been extensively studied in [20] for the term of ω and displays chaotic behavior for each $0 \le \omega < 1$. In this study, a control input $u_d(t) \in R$ is introduced in the differential equation for the second state. To generally describe the UCS in the true physical world, the controlled system is assumed to be subjected to external disturbance bounded by $|\rho(t)| \le \gamma$, γ is a known positive constant. Thus the controlled UCS in matrix form becomes

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 10 + 25\omega & 10 + 25\omega & 0 \\ 28 - 35\omega & \omega - 1 & 0 \\ 0 & 0 & -\frac{8}{3} - \frac{1}{3}\omega \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -x_1(t)x_3(t) + \rho(t) + u_d(t) \\ x_1(t)x_2(t) \end{bmatrix}. \tag{4}$$

Suppose that one wants to design a discrete controller on a hybrid system and assumes that it is implemented by using zero-order hold device. In this way, the digitally implemented controller $u_d(t)$ is piecewise-constant, that is $u_d(t) = u_d(kT)$ for $kT \le t < (k+1)T$ where $T > 0$ is the sample time period. A realization of such hybrid controlled system is presented in Fig. 1. According to (2), the discrete-time dynamics of UCS with $\omega = 0$ and sample time $T = 0.001$ s can be obtained as

$$\begin{aligned}
x_d(k+1)T &= \begin{bmatrix} x_{d1}(k+1)T \\ x_{d2}(k+1)T \\ x_{d3}(k+1)T \end{bmatrix} = \begin{bmatrix} 0.990 & 0.010 & 0 \\ 0.028 & 0.999 & 0 \\ 0 & 0 & 0.997 \end{bmatrix} x(kT) \\
&\quad + \begin{bmatrix} 0 & 0 \\ 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \begin{bmatrix} -x_{d1}(kT)x_{d3}(kT) + u_d(kT) + \rho(kT) \\ x_{d1}(kT)x_{d2}(kT) \end{bmatrix}.
\end{aligned} \tag{5}$$

Obviously, for the UCS with different values of $0 \le \omega < 1$, the discrete-time models can be also easily obtained by (2). In the following, we propose the design procedure for the case of $\omega = 0$, however it can be easily applied for UCS with all $0 \le \omega < 1$. As mentioned above, the aim of this work is to propose a discrete control law $u_d(kT)$ such that the chaotic behavior of continuous UCS can be robustly suppressed. Accordingly, to achieve the control goal based on the DSMC technique, there exist two basic steps for the design procedure. The first step is to construct an appropriate switching surface such that the sliding motion can result in $\lim_{t\to\infty} \|x_d(t)\| = 0$. The second step is to establish a DSMC law which can guarantee the attraction of the sliding manifold.