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Finite-time stabilization control of memristor-based neural networks ☆

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ABSTRACT

This paper investigates the finite-time stabilization problem for a general class of memristor-based neural networks (MNNs). Firstly, based on set-valued analysis and Kakutani's fixed point theorem of set-valued maps, the existence of equilibrium point can be guaranteed for MNNs. Then, by designing novel discontinuous controller, some sufficient conditions are proposed to stabilize the states of such MNNs in finite time. Moreover, we give the upper bound of the settling time for stabilization which depends on the system parameters and control gains. The main tools to be used involve the framework of Filippov differential inclusions, non-smooth analysis, matrix theory and the famous finite-time stability theorem of nonlinear system. Finally, the theoretical results are verified by concrete examples with computer simulations.

1. Introduction

Since the pioneering work of Prof. Chua [1], memristor and its application in neural networks have been extensively studied for a rather long time. As is well known, the memristor-based neural networks (MNNs) can be implemented by very large scale integration (VLSI) circuits and the synaptic connection weights are implemented by the memristors. Actually, the MNNs consist of some identical memristors in a bridge circuit and can be used to perform analog multiplication. Due to the distinctive ability of memristor to memorize the passed quantity of electric charge, the MNNs can remember its past dynamical history. Therefore, in order to emulate the human brain in a real time, the memristor might play a significant role in novel analog circuits of artificial neural network. It should be pointed out that memristor-based circuit networks exhibit complex switching phenomena and the switching rule depends on the state of network. From a mathematical standpoint, the MNNs can be seen as differential equations possessing discontinuous right-hand sides. In recent years, a novel tool named Filippov differential inclusion theory has been introduced to investigate the dynamical behaviors of MNNs such as equilibrium point, periodic orbits, almost periodic orbits, synchronization, convergence of solution in the sense of Filippov,

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and so on [2–14]. However, there is still not much research concerning finite-time stabilization problem of memristor-based neuron system.

As a powerful tool, stabilization control plays an important role in many different science and engineering fields. Especially, in the field of artificial neural networks involving memristor, the study for stabilization is more complicated due to the state-dependent nonlinear switching behaviors of MNNs. There are many types of stabilization including asymptotic stabilization, exponential stabilization, pinning stabilization, finite-time stabilization, etc. Unfortunately, only the exponential or asymptotic stabilization whose convergence time is sufficiently large can be realized for MNNs in most literature [4,9]. Different from exponential or asymptotic stabilization, finite-time stabilization control means that the trajectories of system states can be controlled to approach some equilibrium values in a finite time and to keep them there then after. Thus, the convergence time of finite-time stabilization can be shortened for guaranteeing fast response. On the other hand, the finite-time stabilization could be realized by many control techniques such as classical linear feedback control, adaptive feedback control, sliding mode control, and so on. It is noted that discontinuous controllers are more easy to achieve finite-time stabilization. For example, the authors in [15,16] used discontinuous controllers to deal with the finite-time stabilization problem of neural network models. However, there are very few papers using discontinuous controllers to stabilize memristor-based neural networks in finite time. In addition, because of discontinuous vector field, there still lack suitable and efficacious methods to investigate the finite-time stabilization issues for memristor-based neuron system consisting of too many subsystems. The theoretical and technical difficulties in studying finite-time stabilization problem of MNNs mainly include two aspects: (1) Because of the special discontinuous switching features of memristor, the memristive neural networks are usually described by differential equations with discontinuous right-hand sides. In this case, the classical solutions (i.e., the continuously differential solutions) are not guaranteed for such discontinuous memristive neuron systems and the classical theory of differential equations is shown to be invalid. (2) The classical control method without switching term is difficult to realize the finite-time stabilization of memristor-based neural networks. That is because the uncertain differences between the Filippov solutions of the discontinuous memristive systems consisting of too many subsystems cannot be well handled. Therefore, we will design novel discontinuous controller and introduce the framework of differential inclusion theory given by Filippov to overcome the difficulties of research work concerning finite-time stabilization of MNNs.

Suppose that we use memristors to replace resistors in the circuit realization of neural networks, then it will result in a state-dependent switching dynamical system. In this paper, we consider a general class of memristor-based neural network model whose dynamic behavior can be described by the following differential equation:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t)) + J_i, \quad t \geq 0,\ i \in \mathbb{N}, \tag{1}$$

where $\mathbb{N} = \{1, 2, \ldots, n\}$, n corresponds to the number of units in a neural network; $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$; $f_j(x_j(t))$ denotes feedback function between the jth-dimension of the memristor and $x_i(t)$; J_i is the external input to the ith unit; $d_i > 0$ denotes the charging rate with which the ith neuron unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $a_{ij}(x_i(t))$ is memristor-based connection weight, and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \mathrm{SGN}_{ij}, \qquad \mathrm{SGN}_{ij} = \begin{cases} 1, & \text{if } i \neq j, \\ -1, & \text{if } i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ represents the memductances of memristors $\mathbf{R}_{ij}$. $\mathbf{R}_{ij}$ denotes the memristor between the feedback function $f_j(x_j(t))$ and $x_i(t)$. According to the feature of the memristor and the current–voltage characteristic, the memristor-based connection weight $a_{ij}(x_i(t))$ possesses following discontinuous property:

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & \text{if } |x_i(t)| \leq \mathscr{S}_i, \\ \check{a}_{ij}, & \text{if } |x_i(t)| > \mathscr{S}_i, \end{cases}$$

for $i, j \in \mathbb{N}$, where the switching jumps $\mathscr{S}_i > 0$, $\hat{a}_{ij}$ and $\check{a}_{ij}$ are all constant numbers.

The differential equation system (1) can be transformed into the vector form as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -Dx(t) + A(x(t))f(x(t)) + J, \quad t \geq 0, \tag{2}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$, $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$, $A(x(t)) = \left(a_{ij}(x_i(t))\right)_{n\times n}$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}}$, and $J = (J_1, J_2, \ldots, J_n)^{\mathrm{T}}$.

Throughout this paper, the following basic condition for memristive neuron system (1) will be used:

($\mathscr{H}1$) For $j \in \mathbb{N}$, for all $u, v \in \mathbb{R}$, $u \neq v$, the feedback function f_j is bounded and satisfies Lipschitz condition; that is to say, there exist positive constants L_j, $\mathscr{M}_j (j \in \mathbb{N})$ such that

$$|f_j(u) - f_j(v)| \leq L_j|u - v|, \quad |f_j(\cdot)| \leq \mathscr{M}_j.$$

Under the framework of Filippov solutions, this paper will deal with the finite-time stabilization problem of memristor-based neural network system (1) or (2) which possesses discontinuous switching jumps. The rest of this paper is organized