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Ambiguous relative orbits in sequential relative orbit estimation with range-only measurements

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ABSTRACT

This paper describes a manifold of ambiguous spacecraft relative orbits that arise in sequential relative orbit estimation. The development herein assumes linear relative dynamics, a circular reference orbit, and range-only measurements. Using a formulation based on relative orbit elements, the ambiguous orbits are categorized into two cases: mirror orbits, which conserve the size and shape but transform the orientation of the true relative orbit, and deformed orbits, which both distort the shape and change the orientation. A special case, that of central ambiguous relative orbits, which are geometrically symmetric relative to the chief's local-vertical-local-horizontal frame is also discussed. The multiplicity of mirror ambiguous orbits, deformed ambiguous orbits and central ambiguous orbits are shown to be three, four and infinity, respectively. Numerical results using an extended Kalman filter are provided to confirm the existence of these ambiguous orbits. Furthermore, the observability is studied analytically with a nonlinear observability criterion using Lie derivatives. It is also shown by numerical results that the inclusion of nonlinearities in the filter model can help resist the tendency of an extended Kalman filter to converge to the ambiguous relative orbits. Finally, the persistence of these ambiguous orbits under unmodeled chief eccentricity error and J2 perturbation is studied.

1. Introduction

Relative orbit estimation is desirable for many types of spacecraft missions including close proximity operations such as formation control, rendezvous, and space-based orbit determination in which relative navigation between spacecraft is required. Among different relative orbit navigation strategies, estimation based only on on-board measurements reduces the total operating cost and improves safety against communication interruptions with ground stations [1–3]. Recently, relative orbit estimation based on linear dynamics and certain simple on-board relative measurements such as angles-only or range-only measurements has received attention in the literature.

In particular, the problem of using angles-only measurements in relative orbit estimation has been thoroughly discussed in the literature. For example, Yim et al. [4] numerically studied the observability of relative orbit estimation by taking line-of-sight (LOS) measurements with the incorporation of J2 perturbation. Woffiden and Geller [5,6] discussed relative orbit estimation based on LOS measurements using a Hill-Clohessy-Wiltshire (HCW) dynamic model [7,8] and discussed the problem of unobservability in this scenario. Kaufman et al. [9] showed that with LOS measurements only, the nonlinear relative orbital dynamics are observable under certain geometric conditions. In contrast, only a few recent papers have dealt with the issue of observability (or lack thereof) using range-only measurements. Rundberg and Lovell [10] discussed the ambiguous relative orbits in initial relative orbit determination (IROD) using a minimal number of range-only measurements. Wang et al. [11] studied the lack of observability of linear dynamics with range-only measurements and also how nonlinearities in the filter model for an extended Kalman filter (EKF) can improve the local observability properties. Since only a limited numerical analysis of the ambiguous orbits using range-only measurements was discussed in this paper (Wang et al. [11]), a second paper, proposed subsequently by Wang et al. [12], focused on a systematic analysis of the ambiguous orbits with range-only measurements using relative orbit elements [13] (ROEs) as well as the effects of higher order nonlinear models on avoiding these ambiguous orbits. At the same time, Christian [14] also developed the explanation and categorization of these ambiguous orbits by the use of initial Cartesian relative coordinates, which was followed by a more comprehensive study on the same topic [15]. Compared to these previous works [10–12,14,15], this new paper offers more details on the analysis of ambiguous orbits, new insights on the special category of central ambiguous orbits, an in-depth analysis of observability

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using the nonlinear observability criteria of Lie derivatives, and a discussion of the persistence of these ambiguous orbits under unmodeled chief eccentricity error and J2 perturbation.

This paper begins with a discussion of the reason for the appearance of ambiguous relative orbits using ROEs. Subsequently, the enumeration and classification of these relative orbits is provided both by using Cartesian coordinates and geometric properties of the relative orbit. The condition for the existence of deformed ambiguous relative orbits is also presented through the solution of a fourth order polynomial. The special case of a central relative orbit, which has infinite number of ambiguous orbits is discussed and shown to be unobservable. Numerical results are given to verify the existence of these ambiguous orbits. In the light of observability analysis, a nonlinear observability criterion using Lie derivatives is adopted to analyze the observability of an estimation scenario using the HCW dynamic model and range-only measurements. As a means to exclude ambiguities, we explore the possibility of using higher order nonlinear models to guarantee the uniqueness of the estimated relative orbit. Finally, the practicality of using these conclusions for ambiguous orbits is studied with unmodeled chief eccentricity error and J2 perturbation.

2. Analysis of ambiguous relative orbits

2.1. Conditions of ambiguous orbits

To formulate the HCW dynamics model, which is appropriate for modeling spacecraft relative motion assuming a circular chief orbit and small separation between the chief and deputy spacecraft, the local-vertical-local-horizontal (LVLH) frame [16] (shown in Fig. 1) is used. In Fig. 1, x denotes the coordinate in the radial direction $\hat{\mathbf{O}}_r$, y denotes the coordinate in the along-track direction $\hat{\mathbf{O}}_\theta$, and z denotes the coordinate in the cross-track direction $\hat{\mathbf{O}}_h$. The equation of relative motion expressed in the chief's LVLH frame can be expressed as

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2x = 0 \\ &\ddot{y} + 2n\dot{x} = 0 \\ &\ddot{z} + n^2z = 0 \end{aligned} \tag{1}$$

where n is the mean motion (angular speed) of the circular chief orbit.

The problem of ambiguous orbits for range measurements was first proposed by Wang et al. [11], in which it is shown that when an EKF based on HCW dynamics and range-only measurements is run, there are instances in which the estimated orbit does not follow or approach the true orbit. Instead, the estimated orbit converges to some other ambiguous orbits orbit, two of which are shown in Fig. 2. The figure represents two separate trials where an EKF was initialized with a particular state vector and processed range measurements from the chief to the deputy spacecraft. The range history of the converged estimated orbits in Fig. 2(a) and (b) both match that of the true orbit under HCW dynamics. Also, it is apparent that the estimated orbit in Fig. 2(a) resembles a mirror image of the true orbit and conserves its shape, while the estimated orbit in Fig. 2(b) is deformed in shape compared with the true orbit. An orbit is defined as an ambiguous orbit of the true orbit by range if it shares the same range history with the true orbit. Namely,

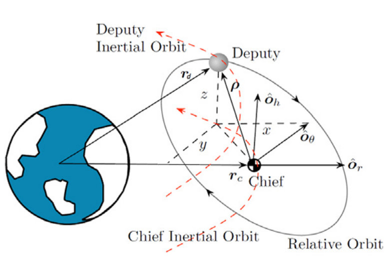

Fig. 1. Rotating LVLH frame used to describe the relative position of the satellites.

$\rho'(t) = \rho(t)$ for all $t > 0$, where $\rho(t)$ and $\rho'(t)$ are the ranges of the true and ambiguous orbits at time t respectively. Furthermore, if the ambiguous orbit conserves the size and shape of the true orbit then it is classified as a mirror ambiguous orbit; otherwise, it is classified as a deformed ambiguous orbit.

The analysis of range-only ambiguous orbits is based on the solution of Eq. (1), which can be expressed in terms of relative orbit elements [13](ROEs) as

$$\begin{aligned} &x(t) = -\tfrac{a_e}{2}\cos(\beta) + x_d \\ &y(t) = a_e\sin(\beta) + y_d \\ &z(t) = z_m\cos(\psi) \end{aligned} \tag{2}$$

where a_e, z_m and x_d are constant and represent the in-plane motion magnitude, the out-of-plane motion magnitude and the drift in radial direction, while $y_d(t) = y_{d0} - \frac{3}{2}nx_dt$, $\beta(t) = \beta_0 + nt$ and $\psi(t) = \psi_0 + nt$ are time dependent and represent the drift in along-track direction, the in-plane phase angle and the out-of-plane phase angle. It is clear that $(a_e, z_m, x_d, y_{d0}, \beta_0, \psi_0)$ are six constants that can be used to represent relative orbits.

In the relative orbit estimation problem, $\mathbf{x} = [x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t)]^T$ denotes the state to be estimated, and the range between the chief and deputy can be expressed as $\rho(t) = \sqrt{x^2(t) + y^2(t) + z^2(t)}$. Using the ROEs, the square of range ρ at time t can be expressed as

$$\begin{aligned} \rho^2(t) &= x^2(t) + y^2(t) + z^2(t) \\ &= \tfrac{5}{8}a_e^2 + \tfrac{1}{2}z_m^2 + y_{d0}^2 \\ &+ \left[-\tfrac{3}{8}a_e^2\cos(2\beta_0) + \tfrac{1}{2}z_m^2\cos(2\psi_0)\right]\cos(2nt) \\ &+ \left[\tfrac{3}{8}a_e^2\sin(2\beta_0) - \tfrac{1}{2}z_m^2\sin(2\psi_0)\right]\sin(2nt) \\ &+ 2a_ey_{d0}\sin(\beta_0)\cos(nt) + 2a_ey_{d0}\cos(\beta_0)\sin(nt) \\ &- 3a_ex_d\sin(\beta_0)nt\cos(nt) - 3a_ex_d\cos(\beta_0)nt\sin(nt) \\ &- 3x_dy_{d0}nt + \tfrac{9}{4}x_d^2t^2 \end{aligned} \tag{3}$$

Note that for one orbit to be an ambiguous orbit of the true orbit by range, its range history $\rho'(t)$ must exactly match that of the true orbit $\rho(t)$, i.e., $\rho'^2(t) = \rho^2(t)$. Since the nine basis functions in Eq. (3) are linearly independent, the following nine equalities must hold for an ambiguous orbit,

$$5a_e^2 + 4z_m^2 + 8y_{d0}^2 = 5a_e'^2 + 4z_m'^2 + 8y_{d0}'^2 \tag{4a}$$

$$3a_e^2\cos(2\beta_0) - 4z_m^2\cos(2\psi_0) = 3a_e'^2\cos(2\beta_0') - 4z_m'^2\cos(2\psi_0') \tag{4b}$$

$$3a_e^2\sin(2\beta_0) - 4z_m^2\sin(2\psi_0) = 3a_e'^2\sin(2\beta_0') - 4z_m'^2\sin(2\psi_0') \tag{4c}$$

$$a_ey_{d0}\sin(\beta_0) = a_e'y_{d0}'\sin(\beta_0') \tag{4d}$$

$$a_ey_{d0}\cos(\beta_0) = a_e'y_{d0}'\cos(\beta_0') \tag{4e}$$

$$a_ex_d\sin(\beta_0) = a_e'x_d'\sin(\beta_0') \tag{4f}$$

$$a_ex_d\cos(\beta_0) = a_e'x_d'\cos(\beta_0') \tag{4g}$$

$$x_dy_{d0} = x_d'y_{d0}' \tag{4h}$$

$$x_d^2 = x_d'^2 \tag{4i}$$

where the primed quantities correspond to the ambiguous orbit. It is noted that if the non-drifting condition $x_d = 0$ is satisfied, Eqs. (4f-i) will vanish. In the following section, we first discuss the ambiguity under the non-drifting condition (Eqs. (4a-e)) and then check the validity of the ambiguity once the non-drifting assumption is violated. First, however, we observe that Eqs. (4d) and (4e) yield

$$|a_ey_{d0}| = |a_e'y_{d0}'| \tag{5}$$