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# Linear time-varying model predictive control of magnetically actuated satellites in elliptic orbits

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ABSTRACT

Three-axis attitude control using only magnetic actuators has been considered a difficult task due to earth's time-varying magnetic field, the inherent under-actuation associated with this task, and constraints on control dipole moments. In this study, a linear time-varying model predictive control approach is applied to magnetically actuated satellites in elliptic low-earth orbits for nadir and inertial pointing. A linear time-varying model predictive control problem is formulated using an augmented state-space model based on small-angle approximations for both nadir and inertial-pointing nonlinear dynamics models. To reduce the on-line computational load, the model predictive control design for an inertial-pointing problem with Laguerre functions is proposed, and exponential data weighting is used to improve the numerically ill-conditioned problem. Nonlinear simulation results demonstrate the effectiveness of the proposed method.

## 1. Introduction

A CubeSat is a type of miniaturized satellite used for education and technology demonstrations that usually has a volume of 1 L (10 *cm* cube, 1U) and a mass of no more than 1.33 *kg*. In general, CubeSats are categorized as picosatellites (0.1–1 *kg*) or nanosatellites (1–10 *kg*) given their production in the forms of 1U, 2U, 3U, and 6U, for instance. They can be distinguished from satellites with larger mass ranges by their relatively short development time, low cost, and their usage of the latest technologies. Therefore, magnetic actuators, also known as magnetorquers, have been used for the attitude control of small satellites in low-earth orbits (LEOs) to realize significant progress in small-scale, inexpensive satellite missions due to their simplicity, low cost, and light weights. Unlike a reaction wheel and control momentum gyro with parts that rotate at high speeds, the magnetic torquer is more reliable given its low risk of failure. This makes magnetic actuators very popular for CubeSats operating in LEO in spite of the low pointing accuracy and slow control, as presented in the results of a global survey of publicly known projects [1]. Thus, there have been many studies of three-axis attitude control methods using only magnetorquers until recently [2–4].

However, despite its merits, three-axis magnetic attitude stabilization differs from conventional attitude control problems. The control torque levels are generated by the interaction (cross product) between the earth's magnetic field and the three orthogonal current-driven coils according to the calculated control input. Because the control torque of magnetic torquers generated by the cross product is structurally singular, it is impossible to produce three independent control torques at each time instant. Thus, a magnetically actuated satellite constitutes a remarkable example of an underactuated system. Moreover, it is considered to be a more difficult problem compared to the conventional control problem due to the earth's time-varying magnetic field and geomagnetic modeling uncertainties.

Spacecraft attitude control adjusts the orientation of a spacecraft body axis with respect to a reference coordinate frame, and attitude control methods using only magnetorquers have focused on nadir pointing, referring to the consistent pointing toward the center of the earth, in a circular orbit. Examples include the linear quadratic regulator (LQR) [5,6], fuzzy control [7], sliding-mode control [8], and dynamic neural controller [9]. Moreover, there has been limited research concentrating on inertial pointing, which consistently observes a specific object that does not move like the stars or the sun. Although state feedback and output feedback controllers have been designed for inertial pointing [10,11], those studies focused on the proof of stability by adapting a time-invariant averaging system.

Given that some missions require an elliptic orbit, in this paper, the equations of motion for nadir and inertial pointing in an elliptic orbit are formulated. In elliptic orbits, the known disturbing pitch torque that is proportional to the eccentricity due to the time-varying angular velocity can make a satellite that would otherwise be stable tumble in a

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circular orbit [12]. Generally, an optimal solution from the model predictive control (MPC) approach can be constructed considering unknown disturbances because the optimization process is performed at every time step.

Thus, MPC is potentially an appropriate attitude controller for nadir and inertial pointing in elliptic orbits with magnetorquers only. It features favorable points such as ability to attenuate geomagnetic field uncertainty, the disturbing pitch torque in elliptic orbits, and environmental disturbances based on the receding horizon control, and to readily include magnetic dipole moment constraints within the optimization scheme. With regard to the attitude control problem of magnetically actuated satellite, linear time-invariant MPC (LTI-MPC) [13,14] techniques have been proposed for nadir pointing in a circular orbit, and a nonlinear MPC (NMPC) [15] scheme was used to solve the satellite rate damping problem in the initial acquisition phase. However, the computational burden to solve the NMPC problem optimally is a major concern for small satellite applications.

To consider the time-varying characteristics of the earth's magnetic field and the computing load of the controller, this paper proposes linear time-varying MPC (LTV-MPC) scheme, as illustrated in earlier work [16], based on small-angle approximations of a nonlinear model for attitude control of a small satellite in an elliptic orbit for both nadir and inertial-pointing cases. In addition, exponential data weighting and Laguerre functions are used with inertial-pointing LTV-MPC techniques to solve the numerical problems and to reduce computational load, respectively, as the prediction horizon increases.

The paper is organized as follows. After the introduction, nonlinear and linear models of magnetically controlled attitude dynamics for nadir and inertial-pointing spacecraft are presented. The next section provides the LTV-MPC law design for nadir pointing along with simulation results. Then the proposed LTV-MPC using Laguerre functions and applying exponential data weighting for inertial pointing is addressed, after which the conclusion to the study is given in last section.

## 2. Mathematical model

### 2.1. Coordinate system

To describe the attitude dynamics, kinematics, and geomagnetic field of a rigid satellite with magnetorquers for nadir and inertial pointing, reference coordinate frames are defined as follows. The coordinate systems are Cartesian, right-handed, and orthogonal. Fig. 1 illustrates presented coordinate systems.

#### 2.1.1. Inertial frame

The inertial frame $F_I$, known as an earth-centered inertia (ECI) frame, has its origin at the earth's center. The $X_I$ axis points toward the direction of the vernal equinox. The $Z_I$ axis coincides with the earth's rotation axis; hence, the $Y_I$ axis can be defined using the right-handed rule.

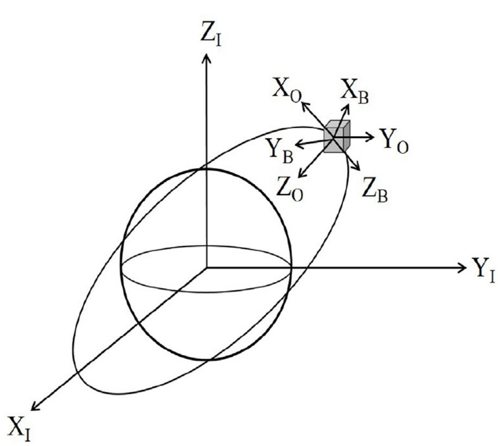


**Fig. 1.** Coordinate systems.

#### 2.1.2. Orbit frame

The orbit frame $F_O$, known as a local-vertical-local-horizontal (LVLH) frame, is attached to the satellite. The $Z_O$ axis points toward the nadir, the $Y_O$ axis follows a negative normal orbit, and the $X_O$ axis points approximately toward the orbital velocity vector.

#### 2.1.3. Body frame

The body frame $F_B$ is fixed at the satellite's center of mass. The body frame is aligned with the principal body axes; that is, the moment of the inertia matrix consists only of principal components. Therefore, the inertia matrix of a satellite in $F_B$ can be written as

$$\mathbf{J} = diag[J_x \quad J_y \quad J_z]. \tag{1}$$

### 2.2. Earth's magnetic field model

The earth's magnetic field model in $F_I$ from earlier work [17] is given such that

$$\mathbf{B}_I(t) = \frac{\mu_m}{\|\mathbf{R}(t)\|^3}[3\{\hat{\mathbf{p}}(t)\cdot\hat{\mathbf{R}}(t)\}\hat{\mathbf{R}}(t) - \hat{\mathbf{p}}(t)], \tag{2}$$

where the field's dipole strength is $\mu_m = 7.746 \times 10^{15} Wb\cdot m$, $\mathbf{R}(t)$ is satellite's position vector, $\hat{\mathbf{R}}(t)$ is the unit vector of $\mathbf{R}(t)$, and $\hat{\mathbf{p}}(t)$ is the unit vector of the geomagnetic dipole expressed in $F_I$:

$$\hat{\mathbf{p}}(t) = \begin{bmatrix} sin\theta_m cos\omega_e t + \alpha_\circ \\ sin\theta_m sin\omega_e t + \alpha_\circ \\ cos\theta_m \end{bmatrix}. \tag{3}$$

Here, the dipole's coelevation is $\theta_m = 170.0°$ for the year 2010, $\omega_e = 360.99°/day$ is the earth's average rotation rate, and $\alpha_0$ is the right ascension of the dipole at time $t = 0$ [11].

The satellite position vector resolved in $F_I$ can be calculated by the integration of the differential equations related to the orbital elements, which can be obtained in the National Aeronautics and Space Administration/North American Aerospace Defense Command (NASA/NORAD) two-line element (TLE) format.

### 2.3. System model for nadir pointing

#### 2.3.1. Kinematics

Kinematic equations describe the rigid body orientation, and Euler angles are used throughout this paper. Spacecraft dynamical equations of motion require transformation between different frames. The direction cosine matrix (DCM) is represented using the transformation matrix $\mathbf{C}_{A/B}$, which transforms from frame $F_B$ to frame $F_A$. In the case of a nadir-pointing mission, the orientation of the satellite is defined in $F_B$ with regard to $F_O$ because $F_B$ is aligned with $F_O$ to satisfy the objective of nadir pointing. With the order of axes transformation [18] as the yaw ($\psi$), pitch ($\theta$), and roll ($\phi$) angle (with the order of rotation of the axes of 3-2-1), the transformation matrix $\mathbf{C}_{B/O}$ is given as

$$\mathbf{C}_{B/O} = \begin{bmatrix} cos\theta cos\psi & cos\theta sin\psi & -sin\theta \\ -cos\phi sin\psi + sin\phi sin\theta cos\psi & cos\phi cos\psi + sin\phi sin\psi sin\theta & sin\phi cos\theta \\ sin\phi sin\psi + cos\phi sin\theta cos\psi & -sin\phi cos\psi + cos\phi sin\psi sin\theta & cos\phi cos\theta \end{bmatrix}, \tag{4}$$

where the Euler angles are the angles between $F_B$ and $F_O$.

The angular velocity vector of the body $\omega^B_{B/O}$, which represents the body angular velocity relative to $F_O$ expressed in $F_B$, can be expressed with the relationships between the Euler angles and their derivatives [18].