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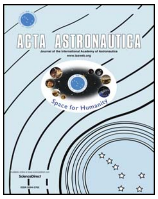

Trajectory optimization for lunar rover performing vertical takeoff vertical landing maneuvers in the presence of terrain

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ABSTRACT

This study presents a trajectory optimization framework for lunar rover performing vertical takeoff vertical landing (VTVL) maneuvers in the presence of terrain using variable-thrust propulsion. First, a VTVL trajectory optimization problem with three-dimensional kinematics and dynamics model, boundary conditions, and path constraints is formulated. Then, a finite-element approach transcribes the formulated trajectory optimization problem into a nonlinear programming (NLP) problem solved by a highly efficient NLP solver. A homotopy-based backtracking strategy is applied to enhance the convergence in solving the formulated VTVL trajectory optimization problem. The optimal thrust solution typically has a "bang-bang" profile considering that bounds are imposed on the magnitude of engine thrust. An adaptive mesh refinement strategy based on a constant Hamiltonian profile is designed to address the difficulty in locating the breakpoints in the thrust profile. Four scenarios are simulated. Simulation results indicate that the proposed trajectory optimization framework has sufficient adaptability to handle VTVL missions efficiently.

1. Introduction

Recently, the exploration of the moon, the nearest celestial body to the earth, has become increasingly attractive for space scientists due to several reasons. For example, Helium-3, which is used in nuclear fusion and can be a future energy source, is abundant on the moon. The existence of water has also been confirmed by the National Aeronautics and Space Administration's (NASA) Lunar Crater Observation and Sensing Satellite. Furthermore, the moon can be an advance base for exploring other planets [1,2]. Given the potential benefits of lunar exploration, Americans, Europeans, Japanese, Chinese, and Indians are planning to return to the moon [3]. The range of lunar explorations is expected to increase in the future. Approximately 83% of the moon's surface is composed of lunar highlands [4]. Hence, lunar rovers (LRs) can explore areas with complex terrains. Vertical takeoff vertical landing (VTVL) LR has recently become the focus of studies. VTVL LR executes "hopping" maneuvers using the reverse force of the propeller to take off vertically, translate a desired distance, and land vertically [5]. Compared with a wheeled rover, a VTVL "hopping" rover has the advantage to cross terrain which would be impassable to most wheeled rovers such as craters and boulder fields.

In previous VTVL vehicles, Surveyor 6 performed a lateral 2.4 m hop to investigate the surface mechanical properties of lunar regolith in 1967, which aided in determining whether the surface was suitable for manned missions. The Hayabusa mission included a hopping robot called Minerva, which was designed to bounce along the surface of the asteroid Itokawa. However, Minerva was accidentally deployed faster than the Itokawa's escape velocity upon its arrival in 2005. Minerva survived for several hours but never reached the surface [5]. In the development of current VTVL vehicles, NASA Marshall Spaceflight Center developed a VTVL lunar lander technology demonstrator named Mighty Eagle. Penn State University developed a VTVL lunar lander software demonstrator named Puma [5]. Private aerospace companies, such as SpaceX and Blue Origin, have made remarkable progress in the technology of VTVL reusable rockets [6]. Trajectory design should be considered for vehicles to perform VTVL translational "hopping" maneuvers. Although a large number of studies have analyzed the trajectory design problems of descent [7–13] and ascent [14–18] on the moon for LR, few studies on the trajectory design problem for VTVL translational maneuvers on the moon have been conducted. Ref. [5] analyzed the optimal rocket-powered translational VTVL trajectories with two-dimensional dynamics. VTVL translational "hopping"

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maneuvers are performed over the surface of a body with significant gravity and minimal air resistance, such as the moon or Mars. Ref. [19] developed a trajectory optimization method based on convex optimization approach for pinpoint hopping movement on a small body to improve the stability and accuracy of surface exploration considering the irregular and weak gravity fields near a small body. The present study focuses on trajectory optimization for LR performing three-dimensional VTVL translational "hopping" maneuvers over the surface of the moon with variable-thrust propulsion considering terrain collision avoidance.

Numerical methods on trajectory optimization problem are categorized into indirect and direct methods [20]. Indirect methods have several disadvantages, including a small convergence region, the analytical derivation of the Hamiltonian boundary value problem, a nonintuitive initial guess for the costate, and a priori knowledge of constrained and unconstrained arcs when path constraints are present [21]. Moreover, most trajectory optimization problems do not have an analytical solution. The key of trajectory optimization strategies is the need to determine optimal trajectories for complex system models with efficient and reliable nonlinear programming (NLP) methods [22], which illustrates that direct methods are preferable to solve complex trajectory optimization problems. Pseudospectral methods [21,23–29], which are a class of direct methods where the trajectory optimization problem is transcribed into an NLP problem, have recently become increasingly well-known and widely used to obtain the numerical solution on trajectory optimization problems. The state and control variables are parameterized using global polynomials at collocation nodes derived from a Gaussian quadrature. For problems with smooth solutions, the application of global polynomials associated with Gaussian quadrature collocation points provides accurate approximations and exponential convergence [29]. However, for problems with unsmooth solutions or not well approximated by global polynomials, finite-element approaches [27,30,31] are preferable, where the time interval is partitioned into subintervals, and polynomials are used to approximate the state and control profiles over each subinterval.

The present work proposes a trajectory optimization framework for VTVL optimization problem to be able to handle future VTVL missions for LR. First, the VTVL trajectory optimization problem is established. The terrain description is directly placed in the trajectory optimization problem for collision avoidance. A finite-element approach is selected to transcribe the original dynamic optimization problem into a finite-dimensional NLP problem, which usually has a large-scale dimension. To overcome difficulties in solving the resulting NLP problem, we utilize a homotopy-based backtracking initial value strategy [17,18] to enhance convergence. The thrust of the LR is throttleable and bounded in this study, which always results in the optimal thrust profile typically having a "bang-bang" profile, that is, the thrust magnitude "bangs" instantaneously between its maximum and minimum magnitudes [12,13,32]. Thus, breakpoints exist in the thrust profile. Direct methods are challenged by the need to capture the discontinuities (breakpoints) in control profiles accurately [22]. On the basis of the proposed adaptive mesh refinement strategy in Refs. [12,13], an enhanced adaptive mesh refinement strategy associated with finite-element approaches is designed to address the difficulty in locating the breakpoints in the thrust profile.

The remainder of this paper is organized as follows. Section 2 establishes the VTVL trajectory optimization problem. Section 3 introduces the proposed trajectory optimization framework for VTVL. Section 4 presents the numerical results and discussions. Section 5 provides the conclusion.

2. Problem formulation

The kinematics and dynamics model of VTVL process, where the LR is modeled as a mass point subject to three-degree-of-freedom translational motion, is described with the assumption that the moon is a regular spherical body and the influences of its rotation and other celestial bodies on the LR are ignored. The LR's rocket motor produces a thrust vector that is assumed to be aligned with the LR's longitudinal axis. The thrust vector direction is used as a surrogate for the LR's attitude, thereby maintaining the translational nature of the problem.

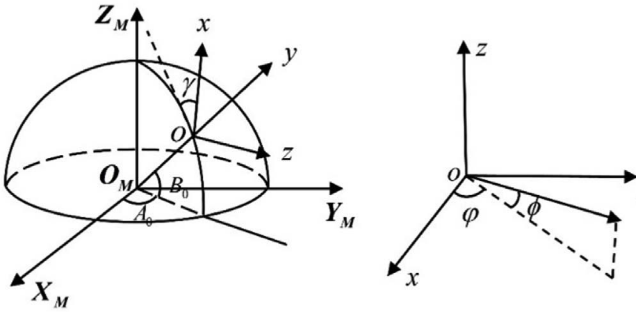

Fig. 1. Coordinates of vertical takeoff vertical landing.

2.1. Kinematics and dynamics model

The VTVL process can be treated in a two-body system. The motion of VTVL is described in three-dimensional coordinates, as shown in Fig. 1. This study assumes that $O_M X_M Y_M Z_M$ and $oxyz$ are the lunar central inertial and lunar launch inertial coordinates, respectively. The kinematics and dynamics equations of the VTVL process in the lunar launch inertial coordinate are expressed as follows [12]:

$$\begin{aligned}
&\frac{dx}{dt} = V_x,\ \frac{dy}{dt} = V_y,\ \frac{dz}{dt} = V_z,\\
&\frac{dV_x}{dt} = \frac{T}{m}\cos\varphi\cos\phi - \frac{\mu x}{(x^2+y^2+z^2)^{3/2}},\\
&\frac{dV_y}{dt} = \frac{T}{m}\sin\varphi\cos\phi - \frac{\mu y}{(x^2+y^2+z^2)^{3/2}},\\
&\frac{dV_z}{dt} = -\frac{T}{m}\sin\phi - \frac{\mu z}{(x^2+y^2+z^2)^{3/2}},\\
&\frac{dm}{dt} = -\frac{T}{I_{sp}g_0},
\end{aligned} \tag{1}$$

where (x, y, z) is the position vector of the LR; (V_x, V_y, V_z) is the velocity vector of the LR; m is the mass of the LR; φ and ϕ represent the pitch and yaw angles, respectively; T denotes the thrust magnitude; I_{sp} is the specific impulse; and μ and g_0 denote the moon's gravitational constant and the Earth's gravitational acceleration, respectively.

2.2. Path constraints

The thrust is throttleable in full range from the lower value to the upper value in this study, and the thrust constraint is expressed as follows:

$$T_{min} \le T \le T_{max}, \tag{2}$$

where T_{min} and T_{max} are the minimum and maximum thrust values, respectively. To avoid sudden angle changes during flight, the following constraints are applied to satisfy the angular rates for the pitch and yaw angles:

$$|d\varphi/dt| \le \omega_{\varphi max},\ |d\phi/dt| \le \omega_{\phi max}, \tag{3}$$

where $\omega_{\varphi max}$ and $\omega_{\phi max}$ are the maximum angular rates of the pitch and yaw angles, respectively. Given that the flight is over the moon's surface, the following expression can be derived:

$$\sqrt{x^2+y^2+z^2} \ge R_M, \tag{4}$$

where R_M is the moon's radius.

For collision avoidance, the highland with cone surface in Fig. 2 is considered at the flight direction of the LR. The cone-surface highland is used to characterize the lunar peaks in a simplified manner. H