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A simple orbit-attitude coupled modelling method for large solar power satellites

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ABSTRACT

A simple modelling method is proposed to study the orbit-attitude coupled dynamics of large solar power satellites based on natural coordinate formulation. The generalized coordinates are composed of Cartesian coordinates of two points and Cartesian components of two unitary vectors instead of Euler angles and angular velocities, which is the reason for its simplicity. Firstly, in order to develop natural coordinate formulation to take gravitational force and gravity gradient torque of a rigid body into account, Taylor series expansion is adopted to approximate the gravitational potential energy. The equations of motion are constructed through constrained Hamilton's equations. Then, an energy- and constraint-conserving algorithm is presented to solve the differential-algebraic equations. Finally, the proposed method is applied to simulate the orbit-attitude coupled dynamics and control of a large solar power satellite considering gravity gradient torque and solar radiation pressure. This method is also applicable to dynamic modelling of other rigid multibody aerospace systems.

1. Introduction

The focus of this paper is to investigate very large solar power satellites (SPSs) that collect solar energy to generate electricity in space and then transmit it to the Earth. Due to the reducing resources and environmental problems of fossil fuel [1], SPSs have attracted much attention from scientists [2]. Since the first concept of SPS was proposed [3], many concepts have been put forward, such as 1979 SPS reference system [1], sail tower SPS [4], tethered SPS [5,39], integrated symmetrical concentrator (ISC) [6], and so on. The concept of ISC can avoid the use of slip rings and long distance power delivery that appear in other concepts [7]. The concept of ISC is that, by siting the primary reflectors at the ends of a long truss and reflecting solar radiation to the solar panel, solar power at high intensity is collected, and then the generated electricity is transmitted to the ground by transmitting antenna. Based on the concept of ISC, Japan Aerospace Exploration Agency (JAXA) has developed several concepts of SPS, such as 2001 JAXA reference model [8], 2002 JAXA reference model [8] and formation flying SPS model [9].

Since an SPS is a very large space system, its dynamics and control are of great importance. However, there are few investigations into the dynamics and control issues of SPSs [10]. McNally et al. [11] studied the orbit dynamics of SPSs in geosynchronous Laplace plane (GLP) orbit and geosynchronous equatorial orbit (GEO), and they found that SPSs located in GLP orbit required almost no fuel to maintain its orbit and could minimize the risk of debris, compared with SPSs in GEO. Wie and Roithmayr [12,13] investigated the effects of perturbations on orbit and attitude dynamics of Abacus SPS, and they designed orbit and attitude controllers considering perturbations and system uncertainties using electric propulsion thrusters. Wu et al. [10] proposed a time-varying robust optimal control strategy and applied it to the attitude control of Abacus SPS. Liu et al. [14] studied the effects of fourth order gravitational force and torque on the dynamic response and control accuracy of the sail tower SPS. Fujii et al. [15,16] investigated the vibration control algorithm for solar panels of tethered SPS by adjusting the tension of tethers, and they verified their method through experiments on the ground. Ishimura and Higuchi [17] studied the coupled dynamics of attitude motion and structural vibration of tethered SPS, and they found that the coupling phenomenon results from low stiffness of tethers and thermal deformation of solar panels. Senda and Goto [18] constructed a dynamic model of tethered SPS and proposed an attitude control method by geomagnetic force. Jin et al. [19,20] studied the trajectory planning for SPSs with reflectors to obtain real-time Earth pointing and Sun

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pointing by rotating the truss and the reflectors cooperatively.

From the aforementioned review, the Euler angle representation was used to investigate simple single-rigid-body problems. For complicated rigid multibody systems, such as ISC and sail tower SPS, natural coordinate formulation (NCF) is an effective method to simplify the modelling process [21]. NCF uses two Cartesian coordinate points and two Cartesian unitary vectors as dependent generalized coordinates of a rigid body so that the modelling process is very easy to understand [22]. Meanwhile, by sharing the Cartesian coordinate points by contiguous bodies, NCF reduces the number of joint constraints [21,23]. On the basis of NCF, zhao et al. [24] established the solar sails model and investigated the dynamic behavior of deployment. Based on the NCF, Liu et al. [25], constructed the dynamic model for rigid-flexible satellite system, and they [26] investigated the dynamics and control of a satellite-based robot with six arms. However, it is necessary to mention that, in the above works on NCF, the effect of gravity gradient torque was neglected. Gravity gradient torque is one of the main sources of attitude perturbations for SPSs [12], hence, it is necessary to be taken into account [14].

The objective of this paper is to develop NCF to take gravitational force as well as gravity gradient torque into consideration so that this simple modelling method is applicable to orbit-attitude coupled modelling of complicated SPSs. This paper is organized as follows. The orbit-attitude coupled modelling method for a rigid body is proposed in section 2. In section 3, an energy- and constraint-conserving algorithm for DAEs is presented. A simple example is carried out to validate the proposed modelling method and proposed numerical method in section 4. Section 5 presents dynamic modelling and attitude controller design for 2002 JAXA reference model of SPS. Simulation results are given and discussed in section 6 and conclusions are drawn in the last section.

2. Orbit-attitude coupled modelling method

This section presents the derivation of NCF to take gravitational force and gravity gradient torque of a rigid body into account, which begins with some basic concepts of NCF. In NCF, a rigid body is described in a global inertial coordinate system $O-XYZ$, as shown in Fig. 1. P_i and P_j are two fixed points of the rigid body. $\boldsymbol{e}$, $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal unit vectors connected to the rigid body. $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ are the vectors of global coordinates of P_i and P_j. l is the distance between P_i and P_j. In order to describe the motion of a rigid body, $\boldsymbol{r}_i$, $\boldsymbol{r}_j$, $\boldsymbol{u}$ and $\boldsymbol{v}$ are selected as generalized coordinates

$$\boldsymbol{q}=\left[\boldsymbol{r}_{\mathrm{i}}^{\mathrm{T}},\boldsymbol{r}_{\mathrm{j}}^{\mathrm{T}},\boldsymbol{u}^{\mathrm{T}},\boldsymbol{v}^{\mathrm{T}}\right]^{\mathrm{T}}\in\mathrm{R}^{12}. \tag{1}$$

These generalized coordinates are dependent since there are only 6 degrees of freedom for a rigid body. They are subjected to the following constraints [21]

$$\begin{cases}(\boldsymbol{r}_j-\boldsymbol{r}_i)^{\mathrm{T}}(\boldsymbol{r}_j-\boldsymbol{r}_i)-l^2=0,\\ \boldsymbol{u}^{\mathrm{T}}u-1=0,\\ \boldsymbol{v}^{\mathrm{T}}v-1=0,\\ (\boldsymbol{r}_j-\boldsymbol{r}_i)^{\mathrm{T}}u=0,\\ (\boldsymbol{r}_j-\boldsymbol{r}_i)^{\mathrm{T}}v=0,\\ \boldsymbol{u}^{\mathrm{T}}v=0,\end{cases} \tag{2}$$

which describe in sequence the distance between two points, the lengths of two vectors and the orthogonality between vectors. The above constraints are abbreviated as

$$\boldsymbol{g}(\boldsymbol{q})=0\in\mathrm{R}^6. \tag{3}$$

The equations of motion of the rigid body are constructed by constrained Hamilton's equations. Generally, there are two steps: firstly to obtain the constrained Hamiltonian function, and secondly to calculate the derivatives of the constrained Hamiltonian function with respect to generalized variables. The constrained Hamiltonian function of the rigid body is written as [27]

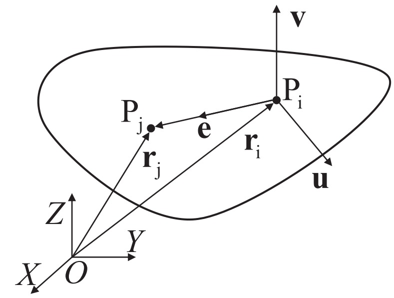

Fig. 1. NCF description of a rigid body.

$$H=T(\dot{\boldsymbol{q}})+U(\boldsymbol{q})+\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{g}(\boldsymbol{q})=\frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{q}}+U(\boldsymbol{q})+\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{g}(\boldsymbol{q}), \tag{4}$$

where $T(\dot{\boldsymbol{q}})$ is the kinetic energy, $\boldsymbol{M}$ is the mass matrix of the rigid body, $U(\boldsymbol{q})$ is the gravitational potential energy and $\boldsymbol{\lambda}\in\mathrm{R}^6$ is the vector of Lagrange multipliers. The mass matrix is calculated by [21]

$$\boldsymbol{M}=\begin{bmatrix}\left(m+\frac{I_x}{l^2}-\frac{2mx_G}{l}\right)\boldsymbol{I}_3 & \left(\frac{mx_G}{l}-\frac{I_x}{l^2}\right)\boldsymbol{I}_3 & \left(my_G-\frac{I_{xy}}{l}\right)\boldsymbol{I}_3 & \left(mz_G-\frac{I_{xz}}{l}\right)\boldsymbol{I}_3\\ & \frac{I_x}{l^2}\boldsymbol{I}_3 & \frac{I_{xy}}{l}\boldsymbol{I}_3 & \frac{I_{xz}}{l}\boldsymbol{I}_3\\ \text{symmetric} & & I_y\boldsymbol{I}_3 & I_{yz}\boldsymbol{I}_3\\ & & & I_z\boldsymbol{I}_3\end{bmatrix}, \tag{5}$$

where $\boldsymbol{I}_3\in\mathrm{R}^{3\times3}$ is an identity matrix, m is the mass of the rigid body, and $[x_G,y_G,z_G]^{\mathrm{T}}$ are the coordinates of centre of mass in $P_i-\boldsymbol{euv}$ coordinate system. I_{xx}, I_{yy}, and I_{zz} are the moments of inertia with respect to $P_i-\boldsymbol{euv}$, I_{xy}, I_{yz} and I_{xz} are the products of inertia with respect to $P_i-\boldsymbol{euv}$. I_x, I_y, and I_z are calculated as

$$\begin{bmatrix}I_x\\I_y\\I_z\end{bmatrix}=\frac{1}{2}\begin{bmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{bmatrix}\begin{bmatrix}I_{xx}\\I_{yy}\\I_{zz}\end{bmatrix}. \tag{6}$$

The gravitational potential energy in Eq. (4) is calculated as [28]

$$U(\boldsymbol{q})=-\mu\int_V\frac{\rho}{\sqrt{\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r}}}\mathrm{d}V=-\mu\int_V\rho f(\boldsymbol{r})\mathrm{d}V, \tag{7}$$

where $\boldsymbol{r}$ is the Cartesian coordinates of an arbitrary point in the rigid body, $f(\boldsymbol{r})=1/\sqrt{\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r}}$ is a nonlinear function of $\boldsymbol{r}$, $\mu=3.986\times10^{14}\ \mathrm{m^3s^{-2}}$ is the standard gravitational parameter of the Earth, ρ is the density of the rigid body, and V is the volume of the rigid body. However, it is not easy to obtain $U(\boldsymbol{q})$ analytically. Wang and Xu employed Taylor series expansion to approximate the gravitational potential energy of a rigid body [28]. According to their results, both the lowest order of gravity gradient torque and second order of gravitational potential energy are expressed by the inertia matrix of the rigid body. Therefore, in order to take gravity gradient torque into account, a second-order Taylor series expansion is adopted to approximate $f(\boldsymbol{r})$ around the centre of mass $\boldsymbol{r}_0$, and the approximated gravitational potential energy is expressed by

$$U(\boldsymbol{q})\approx\frac{-\mu}{2(\boldsymbol{r}_0^{\mathrm{T}}\boldsymbol{r}_0)^{5/2}}\left(d_{xx}\frac{I_x}{l^2}+d_{yy}I_y+d_{zz}I_z+d_{xy}\frac{I_{xy}}{l}+d_{xz}\frac{I_{xz}}{l}+d_{yz}I_{yz}+d_xm\frac{x_G}{l}+d_ymy_G+d_zmz_G+d_0m\right), \tag{8}$$

where