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A new method for solving reachable domain of spacecraft with a single impulse

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ABSTRACT

This paper develops a new approach to solve the reachable domain of a spacecraft with a single maximum available impulse. First, the distance in a chosen direction, started from a given position on the initial orbit, is formulated. Then, its extreme value is solved to obtain the maximum reachable distance in this direction. The envelop of the reachable domain in three-dimensional space is determined by solving the maximum reachable distance in all directions. Four scenarios are analyzed, including three typical scenarios (either the maneuver position or impulse direction is fixed, or both are arbitrary) and a new extended scenario (the maneuver position is restricted to an interval and the impulse direction is arbitrary). Moreover, the symmetry and the boundedness of the reachable domain are discussed in detail. The former is helpful to reduce the numerical computation, while the latter decides the maximum eccentricity of the initial orbit for a maximum available impulse. The numerical simulations verify the effectiveness of the proposed method for solving the reachable domain in all four scenarios. Especially, the reachable domain with a highly elliptical initial orbit can be determined successfully, which remains unsolved in the existing papers.

1. Introduction

The reachable domain (RD) is a set of all positions which are accessible for the spacecraft after maneuvering. With the rapid increase of on-orbit spacecraft, the collision probability of the spacecraft in proximity operations, such as formation flying [1,2], rendezvous [3–5] and on-orbit service [6], has risen significantly. The determination of the RD can provide an important reference for analyzing the collision possibility. After a collision event, the early medium-term evolution of debris clouds can also be modeled by using the RD method [7].

The early concepts of the RD date back to the studies of Beckner [8] and Battin [9]. The former researcher studied the accessible region of ballistic missiles, and the latter investigated the envelope of accessibility for missiles or satellites from a fixed point. Their studies could be applied to rockets or ballistic missiles launched from a fixed point on the surface of the Earth while may not be fit for an on-orbit spacecraft with an initial orbital velocity. Vinh et al. [10] proposed the concept of the RD to study the reachable area of an interceptor after a single impulse maneuver with hyperbolic initial speed, noting that the reachable surface after a short time of flight is an ellipsoid. However, their method could not be applied to the situation with long or free flight time.

Xue et al. [11–13] first analyzed the RD for a spacecraft with a single impulse and free flight time. They analyzed three typical scenarios: 1) the maneuver position is fixed and the impulse direction is arbitrary, 2) the maneuver position is arbitrary and the impulse direction is fixed, 3) the maneuver position and the impulse direction are both arbitrary. The envelope of the RD in their method was viewed as the intersection of the envelope of the ellipsoid and the envelope of the plane. However, the RD they solved was larger than the actual one, on account that their method is over-approximate. Wen et al. [14,15] obtained the accurate RD in all three scenarios by adopting three different methods, respectively. However, when under the situation with a highly elliptical initial orbit (with eccentricity equal to or greater than 0.7), the method in Ref. [15] failed to solve the RD with arbitrary maneuver position and impulse direction. The RD with special types of maneuver has also been studied. Li et al. [16] considered two types of RD with a single fixed-magnitude coplanar impulse and coplanar continuous thrust. Zhang et al. [17] analyzed the RD with a single tangent impulse considering the trajectory safety.

In the three typical scenarios, the maneuver position is either fixed or arbitrary [13–15]. However, another important scenario, where the maneuver position is restricted to an interval and the impulse direction is arbitrary, has not been studied. This scenario is the extended scenario of fixed maneuver position and arbitrary impulse direction. The scenario of fixed maneuver position and arbitrary impulse direction and the scenario

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of arbitrary maneuver position and impulse direction can be regarded as two limiting cases of this scenario. Thus, it has a wider range of applications. For example, the maneuver position of the spacecraft may be restricted to an interval, rather than a fixed point or the whole orbit, when considering the constraint of the ground station communication [18,19].

A new method is developed to solve the RD in the three typical scenarios and the new introduced scenario. First, a position on the initial orbit is chosen and the distance in a given direction started from this position is formulated. Then, the maximum reachable distance in this direction is solved by evaluating the extreme value of this distance. Finally, by solving the maximum reachable distance in all directions, the envelope of the RD in three-dimensional space is obtained. Moreover, the symmetry and the boundedness of the RD are analyzed in detail. Two types of symmetry are found to reduce the computation in numerical simulation. When considering the boundedness of the RD in a Keplerian central gravitational field, the maximum eccentricity of the initial orbit can be determined for a spacecraft with a maximum available impulse. In the scenario of arbitrary maneuver position and impulse direction, the numerical simulations demonstrate that the RD with a highly elliptical initial orbit (with eccentricity equal to or greater than 0.7) can be solved successfully.

2. Problem formulation

First, the inertial frame $OXYZ$ and the orbital frame $Axyz$ used in this paper are introduced (see Fig. 1). The origin of the inertial frame $OXYZ$ is located at the center of the Earth, the X axis points toward the perigee of the initial orbit, the Z axis is perpendicular to the plane of the initial orbit, and the Y axis completes the right-handed orthogonal coordinate frame. The origin of the orbital frame $Axyz$ is located at the center of the spacecraft, the x axis is in the direction from the Earth to the spacecraft, the z axis is perpendicular to the plane of the initial orbit, and the y axis completes the right-handed orthogonal coordinate frame. The frame transformation matrix from the orbital frame $Axyz$ to the inertial frame $OXYZ$ is

$$M_{OA} = \begin{bmatrix} \cos f & -\sin f & 0 \\ \sin f & \cos f & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where f is the true anomaly of the spacecraft position.

The spacecraft is assumed to fly in a Kepelerian central gravitational field with free time. As shown in Fig. 1, the thick solid line represents the initial orbit with semimajor axis a_0 and eccentricity e_0, while the dashed line represents the transfer trajectory. Point C represents a chosen position on the initial orbit with a given true anomaly α. A chosen unit vector l, which is in the plane perpendicular to the XY plane, is represented by $(\alpha,\ \varphi)$, where φ is the angle between l and the XY plane. To determine the maximum reachable distance in a given direction $(\alpha,\ \varphi)$, the maneuver position and the impulse vector need to be found to make the transfer trajectory intersect this direction at a point denoted by B, and to maximize the length of BC.

As shown in Fig. 1, the maneuver position is denoted by A, where the true anomaly is f_0. As shown in Fig. 2, the angle between the impulse vector $\Delta\boldsymbol{v}$ and the xy plane is denoted by κ, and the angle between the projection of $\Delta\boldsymbol{v}$ on the xy plane and the x axis is denoted by λ. Thus, in a given direction $(\alpha,\ \varphi)$, the length of BC, denoted by r_{BC}, is determined by three control variables f_0, λ and κ. Then it is maximized to obtain the maximum reachable distance $r_{BC\ \max}$. As shown in Fig. 3, for any given α, the angle φ can vary over the interval $[0,\ 2\pi)$ due to the geometrical characteristic that the initial orbit is always contained in the RD. By solving the maximum reachable distance in all directions $\{(\alpha,\ \varphi)|\alpha\in[0,\ 2\pi),\ \varphi\in[0,\ 2\pi)\}$, the envelope of the RD in three-dimensional space is determined. The orbit parameters of the transfer trajectory and the length r_{BC} in a given direction $(\alpha,\ \varphi)$ are formulated at the rest of this section, and the work of solving the maximum $r_{BC\ \max}$ is performed later in Section 3.

In the frame $Axyz$, the maneuver position vector $\boldsymbol{r}_0$ and the velocity vector $\boldsymbol{v}_0$ of the spacecraft in the initial orbit are written as

$$(\boldsymbol{r}_0)_A = [r_0 \quad 0 \quad 0]^T \tag{2}$$

$$(\boldsymbol{v}_0)_A = \sqrt{\frac{\mu}{p_0}}[e_0\sin f_0 \quad 1+e_0\cos f_0 \quad 0]^T \tag{3}$$

where $r_0 = p_0/(1+e_0\cos f_0)$, p_0 is the semilatus rectum of the initial orbit, $p_0 = a_0(1-e_0^2)$. The impulse vector $\Delta\boldsymbol{v}$ expressed in the frame $Axyz$ is given as

$$(\Delta\boldsymbol{v})_A = \Delta v[\cos\kappa\cos\lambda \quad \cos\kappa\sin\lambda \quad \sin\kappa]^T \tag{4}$$

where Δv is the maximum available velocity increment under the fuel constraint of the spacecraft.

After maneuvering, the spacecraft enters the transfer trajectory. The initial position $\boldsymbol{r}_1$ and velocity $\boldsymbol{v}_1$ of the transfer trajectory are expressed in the inertial frame $OXYZ$:

$$(\boldsymbol{r}_1)_O = M_{OA}(\boldsymbol{r}_0)_A = [r_0\cos f_0 \quad r_0\sin f_0 \quad 0]^T \tag{5}$$

$$\begin{aligned}(\boldsymbol{v}_1)_O &= \boldsymbol{M}_{OA}\left((\boldsymbol{v}_0)_A + (\Delta\boldsymbol{v})_A\right) \\ &= \begin{bmatrix} -\sqrt{\mu/p_0}\sin f_0 + \Delta v\cos\kappa\cos(\lambda+f_0) \\ \sqrt{\mu/p_0}(e_0+\cos f_0) + \Delta v\cos\kappa\sin(\lambda+f_0) \\ \Delta v\sin\kappa \end{bmatrix}\end{aligned} \tag{6}$$

The angular momentum of the transfer trajectory is calculated by

$$\boldsymbol{h}_1 = \boldsymbol{r}_1 \times \boldsymbol{v}_1 \tag{7}$$

Substituting Eqs. (5) and (6) into Eq. (7), then the resulting expression can be simplified to

$$(\boldsymbol{h}_1)_O = \begin{bmatrix} r_0\Delta v\sin f_0\sin\kappa \\ -r_0\Delta v\cos f_0\sin\kappa \\ h_0 + r_0\Delta v\sin\lambda\cos\kappa \end{bmatrix} \tag{8}$$

and the angular momentum magnitude is also obtained:

$$h_1 = \sqrt{r_0^2\Delta v^2\sin^2\kappa + (h_0 + r_0\Delta v\sin\lambda\cos\kappa)^2} \tag{9}$$

The dihedral angle i between the plane of the initial orbit and that of the transfer trajectory satisfies

$$\cos i = \frac{(\widehat{\boldsymbol{h}}_0)_O\cdot(\boldsymbol{h}_1)_O}{h_1} \tag{10}$$

where $(\widehat{\boldsymbol{h}}_0)_O$ is the unit vector aligned to the angular momentum of the initial orbit, $(\widehat{\boldsymbol{h}}_0)_O = [0,\ 0,\ 1]^T$. Substituting Eq. (8) and $(\widehat{\boldsymbol{h}}_0)_O$ into Eq. (10) yields

$$i = \arccos[(h_0 + r_0\Delta v\sin\lambda\cos\kappa)/h_1] \tag{11}$$

According to the geometrical relationships in a spherical triangle, the angle β between OA and OB satisfies

$$\tan\beta\cos i = \tan(\alpha - f_0) \tag{12}$$

Therefore,

$$\beta = \arctan\left[\frac{\tan(\alpha - f_0)}{\cos i}\right] \tag{13}$$

In the transfer trajectory, the distance r_{OB} can be calculated as

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