Acta Astronautica 145 (2018) 396–407

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Influence of surface error on electromagnetic performance of reflectors based on Zernike polynomials

Tuanjie Li *, Jiachen Shi, Yaqiong Tang

School of Mechano-Electronic Engineering, Xidian University, P.O. Box 188, Xi'an 710071, China

A R T I C L E I N F O

Keywords:
Reflector
Surface error distribution
Zernike polynomials
Electromagnetic performance
Influence rule

A B S T R A C T

This paper investigates the influence of surface error distribution on the electromagnetic performance of antennas. The normalized Zernike polynomials are used to describe a smooth and continuous deformation surface. Based on the geometrical optics and piecewise linear fitting method, the electrical performance of reflector described by the Zernike polynomials is derived to reveal the relationship between surface error distribution and electromagnetic performance. Then the relation database between surface figure and electric performance is built for ideal and deformed surfaces to realize rapidly calculation of far-field electric performances. The simulation analysis of the influence of Zernike polynomials on the electrical properties for the axis-symmetrical reflector with the axial mode helical antenna as feed is further conducted to verify the correctness of the proposed method. Finally, the influence rules of surface error distribution on electromagnetic performance are summarized. The simulation results show that some terms of Zernike polynomials may decrease the amplitude of main lobe of antenna pattern, and some may reduce the pointing accuracy. This work extracts a new concept for reflector's shape adjustment in manufacturing process.

1. Introduction

In recent years, reflector antennas such as cable-net antennas and membrane antennas are widely used in communications, radar and radio astronomy to obtain higher gain, stronger directivity, higher reliability and more efficient data-transmission capability [1]. It is well known that, the surface error is inevitable for reflector antennas. The surface error of reflector antennas will directly impact the far-field radiation performance [2–4], so it is necessary to study and summarize the influence rules of surface error distribution on the antenna's electromagnetic performance.

Many researchers have paid much attention to the influence of surface error on antenna's electromagnetic properties. Hao et al. [5] and Rahmat-Samii [6] adopted the divided-fit method to discretize the aperture plane. They divided reflector into many annular areas, then divided each annular area using the interpolation/fitting method. After this, they studied the influence of each local deformation on the antenna's electromagnetic performance. At last, the whole electromagnetic performance was obtained by summing the radiation properties over each local area. This method is convenient for calculating, but it is unavoidable to introduce the theoretical error. Smith and James [7] researched a complex orthogonal basis function to express the axial error between the distorted reflector and the ideal reflector which made the calculation more complex and the workload heavier. Zhang et al. [8] divided the reflector surface into a series of triangular elements of which the projected triangles in aperture plane are the regular triangles. They pointed out that there were only six triangles around each node, so the displacement of this node was only related to these six triangles. Then the whole electrical performance could be obtained by summing the results of the integration over all nodes. Li and Su [9] proposed the method for analyzing the electrical properties of wire mesh including amplitude difference, phase difference and reflecting loss. Based on the physical optical (PO) method, Rahmat-Samii and Galindo-Israel [10] expressed the error using a vector, and substituted the error vector into the radiation integral formula approximating by Fourier transform, and used Jacobi-Bessel series to calculate the Fourier transformation. It can be found that the present methods can be classified into three categories. The first one is to divide the reflector into many small pieces, and sum the electrical properties of the pieces into that of the whole reflector. The second category is to introduce a complex orthogonal basis function to express the distorted reflector. And the third one is to represent surface error by a vector so that the influence rules of error on electrical properties can be obtained by a series of mathematical transformation.

However, the above-mentioned methods only report the influence

* Corresponding author.
E-mail address: tjli888@126.com (T. Li).

https://doi.org/10.1016/j.actaastro.2018.01.063
Received 2 August 2017; Received in revised form 22 December 2017; Accepted 31 January 2018

Table 1
The relationship between Zernike terms and Seidel aberration.

Aberration	Zernike terms
Piston	Z_0
Tilt	Z_1, Z_2
Defocus	Z_3
Astigmatism and coma	$Z_4 \sim Z_7$, $Z_9 \sim Z_{14}$, $Z_{16} \sim Z_{23}$, $Z_{25} \sim Z_{34}$, $Z_{36} \sim Z_{37}$
Spherical aberration	Z_8, Z_{15}, Z_{24}, Z_{35}

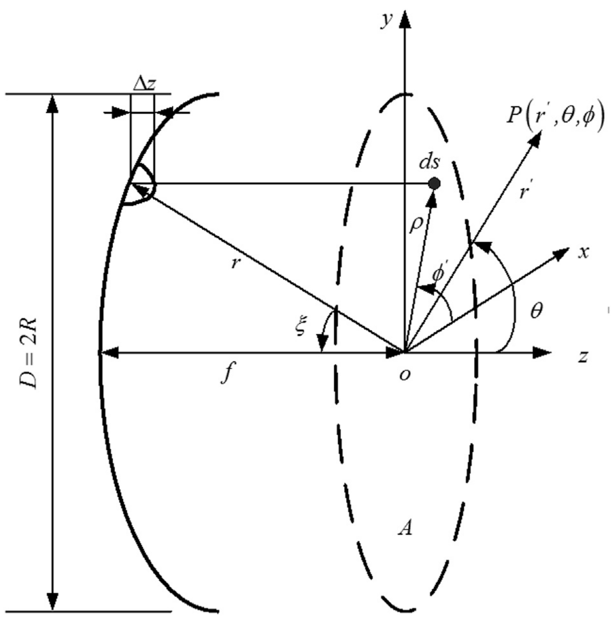

Fig. 1. Geometric schematic of reflector with surface error.

result of surface error on electromagnetic performances, and don't investigate the influence rules of error distribution on electrical properties of reflectors. So they just apply to surface error analysis but not to antenna designing and shape adjustment.

In 1934, Zernike constructed the Zernike polynomials of which the terms were orthogonal in a unit circle [11]. Zernike polynomials have been widely used to describe surface deformation of optical telescopes. They are related to Seidel aberrations such as translation, tilt, defocus, spherical aberration, and coma. Porter et al. [12] applied Zernike polynomials to the wave front aberration fitting of human eye, and obtained the combination of optimal model of interference wave surfaces, which improved the accuracy of wave front fitting. Through the exploration of published researches, it can be found that Zernike polynomials have good astringency and offer more useful information among a number of orthogonal basis functions in the wave front fitting, and they can be applied to fit the smooth and continuous surfaces. Furthermore, their orthogonal property on the unit circle promises each Zernike polynomial is independent of others, which can effectively reduce accidental error.

Therefore, inspired by the optical meaning of Zernike polynomials and their contribution to optical mirror surface researches, this paper uses Zernike polynomials to investigate the electromagnetic properties of reflectors to reveal the influences of error distribution on electrical properties, which will further provide a theoretical basis for antenna design, fabrication and further shape adjustment.

In this paper, the geometrical optics (GO) method is used to deduce the mathematic model between surface error of reflectors and electromagnetic field. First, based on GO method and energy conservation law, the vector distribution of radical field emitted from the feed and reflected by the reflector to aperture plane is calculated. Then the radiation field of aperture plane is evaluated from the electrical field distribution of aperture plane with reflector surface error represented by Zernike polynomials. Compared with the above-mentioned methods, the introduction of Zernike polynomials ensures the continuous smoothness of the fitting reflector surface to make it possible to calculate the far-field electrical performance directly, and changes the optimization variables from z coordinate value of each point to each coefficient of Zernike polynomials, which greatly reduces the number of optimization variables.

The remainder of this paper is organized as follows. In Section 2, the normalized Zernike polynomials are introduced first. Then they are used to describe a smooth and continuous deformation surface. Section 3 presents the formula derivation in detail and gives the mathematic model between the electromagnetic fields and reflector error represented by Zernike polynomials. In Section 4, a simulation analysis of the influences of Zernike polynomials on the electrical properties for the axis-symmetrical reflector with the axial mode helical antenna as feed is further studied. A summary is given in Section 5.

2. Reflector surface error described by Zernike polynomials

2.1. Zernike polynomials

The Zernike polynomials are represented by two variables ρ and θ in polar coordinates. They are weighted orthogonal in a unit circle [13]. Their mathematical form can be expressed as follows [14].

$$\left.\begin{array}{l} Z_{\text{even } t} = [2(n+1)]^{1/2} R_n^m(\rho)\cos m\theta \\ Z_{\text{odd } t} = [2(n+1)]^{1/2} R_n^m(\rho)\sin m\theta \end{array}\right\} \quad m \neq 0$$
$$Z_t = [(n+1)]^{1/2} R_n^m(\rho), \quad m = 0 \tag{1}$$

$$R_n^m(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s![0.5(n+m)-s]![0.5(n-m)-s]!}\rho^{n-2s} \tag{2}$$

where ρ is the polar radius, θ is the azimuth, t is the number of polynomial terms, m is the azimuthal frequency, and n is the radial degree. Here, n and m are non-negative integers which satisfy $m \le n$, and $n - m =$ even.

In this paper, the Zernike polynomials are chosen to express the error due to the following properties superior to other orthogonal polynomials sets.

1) The Zernike polynomials are easy to build their relationships with Seidel aberration function, and the corresponding relationships are listed in Table 1.
2) The Zernike polynomials are orthogonal [15] which makes each term be independent of others.

2.2. Fitting method of reflector surface error

The discrete deformation data of the reflector surface can be obtained by experimental measurement or mechanics analysis. According to these discrete deformation data, the coefficients of the Zernike polynomials can be optimized to describe the smooth and continuous deformation surface.

In this paper, a cylindrical coordinate system is established to describe the reflector antenna system of which the coordinate origin is set to the situation of the feed. In this cylindrical coordinate system, the coordinate of a certain point on the reflector surface projecting on the aperture plane is (ρ_0, ϕ_0, z_0). To satisfy the orthogonality, the coordinate should be normalized by

$$\begin{cases} \rho_i = \rho_0 / R_a \\ \phi_i' = \phi_0 \\ z_i = z_0 \end{cases} \tag{3}$$

where $R_a = D/2 + D/200$ is the normalization factor, D is the diameter of reflector aperture plane, ρ_i and ϕ_i' $i = 1, 2 \cdots p$ are the normalized polar