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Letter

Experimental study on interaction between a positive mass and a negative effective mass through a mass–spring system

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HIGHLIGHTS

- We investigate the dynamics of a positive mass and a negative mass chain, which shows the two bodies may be self-accelerated in same direction.
- We examine the self-accelerated motion of equivalent two-mass chain in experiment.
- We show that the Hamilton's principle is applicable when the energy of the negative mass unit is properly characterized.

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ABSTRACT

We investigate the interaction between a positive mass and a negative effective mass through a three-mass chain connected with elastic springs, a pair of masses is designed to have an effective negative mass, and it interacts with the third positive one as if an equivalent two-mass chain. The dynamics of the equivalent two-mass chain shows that the two bodies may be self-accelerated in same direction when the effective mass becomes negative, the experiment is also conducted to demonstrate this type of motion. We further show that the energy principle (Hamilton's principle) is applicable if the energy of the negative mass unit is properly characterized. The result may be relevant to composite with cells of effective negative mass, their interaction with matrix may lead to more richer unexpected macroscopic responses.

Dynamics of two bodies connected by a linear spring, predicted by Newton's second and third laws, may be more diverse than that observed in our daily life, i.e., the only breathing-like vibration mode of the two bodies. The other predicted possible motion like self-acceleration in the same direction is strictly forbidden due to the requirement on positivity of mass and stiffness. Hypothetically, the diametric drive, i.e., two bodies are self-accelerated in the same direction being considered as a possible mechanism of space propulsion, is possible only if one mass becomes negative [1] (here the case of negative stiffness is not considered, although it is also possible). It is known that composite material endowed with microstructure may manifest effectively as a body with a negative effective mass [2–4] or even with an anisotropic mass density [5,6] in some frequency ranges. These composites with unusual physical property are now termed as metamaterials, they were recognized as rapid development in the last decade [7–10]. The metamaterial technique may provide a platform to investigate the phenomena related to negative mass, as already being done for example negative effective mass for wave blocking [11,12], zero effective mass for complete transmission without phase change [3] and many others. These phenomena are only discussed with homogenized metamaterials, here we want to go a step further by examining the interaction between a positive mass and a negative (effective) one, which is also relevant to composites with inclusion of negative mass. To this end, we should first realize a negative effective mass and then examine its interaction with a positive one. The first experimental investigation of this kind is conducted by Wimmer et al. [13], they examined the interaction of two photons through a junction formed by two optic fiber loops of different sizes, the phenomenon of the diametric drive acceleration is clearly demonstrated.

In this letter, we will investigate the interaction of a positive mass and a negative one through a simple mass–spring system. The idea is the following: we consider a three-mass chain, one adjacent pair of the masses is designed to behave as a body with

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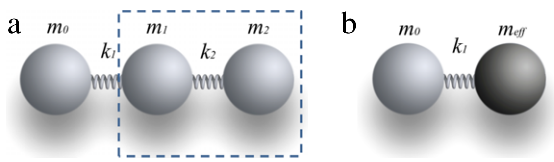

Fig. 1. A three-mass chain (a) and its equivalent two-mass system (b).

equivalent negative mass, and the third positive mass interacts with this equivalent negative mass through an elastic spring. Both theoretical analysis and experiment are performed to illustrate the possible motion mode of the diametric drive. We will also examine whether Hamilton's principle still holds for a system with an effective negative mass concept.

Consider a three-mass chain, as shown in Fig. 1(a), both the mass m_i and stiffness k_j ($i = 0, 1, 2$ and $j = 1, 2$) are positive. The mass 2 is connected with the mass 1, under harmonic motion, these two masses may be considered as an effective mass connected to the mass 0, as shown in Fig. 1(b).

Using Newton's second law, the equations of motion for the three masses are written respectively as

$$m_0\ddot{x}_0 = k_1(x_1 - x_0), \tag{1a}$$

$$m_1\ddot{x}_1 = -k_1(x_1 - x_0) + k_2(x_2 - x_1), \tag{1b}$$

$$m_2\ddot{x}_2 = -k_2(x_2 - x_1), \tag{1c}$$

where $x_i(t)$ is the displacement of the mass i. If we consider a steady harmonic motion $e^{-i\omega t}$ and consider m_2 as a hidden mass, we eliminate x_2 from the equations, this leads to

$$m_0\ddot{x}_0 = k_1(x_1 - x_0), \tag{2a}$$

$$m_{\text{eff}}\ddot{x}_1 = -k_1(x_1 - x_0), \tag{2b}$$

where $m_{\text{eff}} = m_1 + m_2/(1 - \omega^2/\omega_2^2)$, and $\omega_2 = \sqrt{k_2/m_2}$.

Eqs. (2a) and (2b) are exactly the same for a two-mass system, i.e., m_0 and m_{eff} are connected by the spring k_1. Therefore we can tune the frequency ω to let $m_{\text{eff}} < 0$, and examine in turn its interaction with the positive mass m_0. For a general two-mass system with one negative mass, Newton's second law tells that depending on the absolute value of the negative mass, the system will oscillate if it is larger than the positive one and accelerate in one direction otherwise. For the system shown in Fig. 1, it has two natural frequencies, satisfying the following equation

$$\bar{\omega}^4 - (\omega_1^2 + \omega_2^2 + \alpha^2)\bar{\omega}^2 + \omega_1^2\omega_2^2\beta = 0, \tag{3}$$

where $\omega_1 = \sqrt{k_1/m_1}$, $\alpha^2 = k_2/m_1 + k_1/m_0$ and $\beta = (m_0 + m_1 + m_2)/m_0$. So the two natural frequencies are given by

$$\bar{\omega}_{1,2}^2 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 + \alpha^2 \pm \sqrt{(\omega_1^2 + \omega_2^2 + \alpha^2)^2 - 4\omega_1^2\omega_2^2\beta}\right]. \tag{4}$$

At the lower nature frequency $\bar{\omega}_1$, the hidden mass m_2 moves out of phase with respect to the mass m_1, leading to that the pair of the masses m_1 and m_2 behaviors as an effective negative mass, i.e., $m_{\text{eff}}(\bar{\omega}_1) < 0$. In addition, $|m_{\text{eff}}(\bar{\omega}_1)| \geq m_0$ always holds for the three-mass system, meaning that only oscillation motion can be observed when $m_{\text{eff}}(\bar{\omega}_1) < 0$. The eigenvector (vibration mode) at this frequency can be obtained as, $\Phi = [A, B, C]^{\text{T}}$, with $A = m_1\bar{\omega}_1^2/(m_0\omega_2^2) - \beta + 1$, $B = 1 - \bar{\omega}_1^2/\omega_2^2$, and $C = 1$, which will be used as a guide on setting initial condition of the three-mass chain in the following experiment.

For illustration, we will examine the dynamics of a positive mass connected with an effective negative mass by taking $m_0 = 45.83$ g, $m_1 = 55.93$ g, $m_2 = 132.34$ g and $k_1 = k_2 = 34$ N/m. The initial conditions are applied by following the deformation mode at the natural frequency $\bar{\omega}_1 = 21.3$ rad/s, those are $A = -1.957$, $B = -0.763$, $C = 1$, which will trigger a steady harmonic vibration at the frequency $\bar{\omega}_1$. The effective mass of the mass-pair (m_1 and m_2) as function of frequency is given in Fig. 2(a), and at the natural frequency $\bar{\omega}_1$, we find that $m_{\text{eff}} = -127.72$ g. The computed accelerations of the two masses (m_0 and m_{eff}) as function of time at the natural frequency $\bar{\omega}_1$ are shown in Fig. 2(b), it is clearly shown that the two masses are self-accelerated in the same direction (Fig. 2(b)).

In experiment, a three-mass chain connected with elastic springs is mounted on a guided line track, and the mass blocks are lifted by pressured air in order to eliminate friction. First the system is placed free of force in a static equilibrium state, then the mass block m_1 is fixed temporally, and the m_0 and m_2 are displaced with initial displacements $N(A - B)$ and $N(B - C)$, respectively, N is a real value. Finally the system is relaxed simultaneously to move in a free motion state. The displacements of the three mass blocks are measured with laser displacement sensors, the principle of measurement and experimental set-up are shown in Fig. 3(a), (b), respectively. In the experiment, the same parameters as in the simulation are used, so the system vibrates at its natural frequency $\bar{\omega}_1 = 21.3$ rad/s, i.e., 3.4 Hz, and in the experiment we take $N = 5$. The measured displacements of m_0 and m_{eff} are shown in Fig. 3(c), and they are also compared with the model prediction. It is found in deed that the self-acceleration in same direction of the two masses predicted by Newton's second law of motion can take place if the effective mass becomes negative. The experiment results agree quite well with the model's prediction. Figure 3(d) illustrates also the measured force on m_{eff} as function of the measured acceleration, the force is derived by the measured elongation of the spring connecting m_0 and m_{eff}, a linear Newton's second law with the slope m_{eff} is recovered, which also agrees with the model's prediction. Therefore, with the negative effective mass concept we demonstrate experimentally that the self-acceleration motion, well predicted by Newton's second law of motion, can indeed take place.

It is well known that the equation of motion for a multi-body system can also be derived from Hamilton's energy principle, so the question to be addressed in the following is whether the same principle applies if the effective mass becomes negative.

We still consider the previous three-mass chain subjected to a harmonic motion $e^{-i\omega t}$, so the displacement, velocity and acceleration of the mass i are written as

$$X_i = x_i e^{-i\omega t} \tag{5a}$$

$$V_i = v_i e^{-i\omega t} \tag{5b}$$

$$A_i = a_i e^{-i\omega t}. \tag{5c}$$

According to [14], the kinetic and potential energies of the negative effective mass unit are given by $T = (2m_{\text{eff}} + \omega m'_{\text{eff}})V_1^2/4$ and $U = m'_{\text{eff}}\omega^3 X_1^2/4$, with the prime denoting the frequency derivative. These energies are always positive even when the effective mass becomes negative. So the total kinetic and potential energies of the two-mass system with the positive mass m_0 and the effective mass m_{eff} are given by

$$T = \frac{1}{2}m_0V_0^2 + \frac{1}{4}(2m_{\text{eff}} + \omega m'_{\text{eff}})V_1^2 = \frac{1}{2}m_0V_0^2 + \frac{1}{2}m_{\text{eff}}V_1^2 + \frac{1}{4}\omega m'_{\text{eff}}V_1^2, \tag{6a}$$

$$U = \frac{1}{2}k_1(X_1 - X_0)^2 + \frac{1}{4}m'_{\text{eff}}\omega.^3X_1^2. \tag{6b}$$

Lagrangian of the system is now written as

$$L = T - U = \frac{1}{2}m_0V_0^2 + \frac{1}{4}(2m_{\text{eff}} + \omega m'_{\text{eff}})V_1^2 - \frac{1}{2}k_1(X_1 - X_0)^2 - \frac{1}{4}m'_{\text{eff}}\omega^3X_1^2. \tag{7}$$