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Letter

2D hierarchical lattices' imperfection sensitivity to missing bars defect

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ABSTRACT

Commercially available lattices contain various kinds of morphological imperfections which result in great degradation in lattices' mechanical properties, therefore, to obtain imperfection insensitive lattice structure is obviously a practical research subject. Hierarchical structure materials were found to be a class of promising anti-defect materials. This paper builds hierarchical lattice by adding soft adhesion to lattice's cell edges and numerical results show that its imperfection sensitivity to missing bars is minor compared with the classic lattice. Soft adhesion with appropriate properties reinforce cell edge's bending stiffness and thus reduce the bending deformation in lattice caused by missing bars defect, which is confirmed by statistical analysis of normalized node displacements of imperfect lattices under hydrostatic compression and shear loads.

Due to the high specific strength and specific moduli lattice materials have had a wide range of application in recent years [1–3]. However, commercially available lattices always contain defects which result in degradation in lattices' mechanical properties [2–5]. Defects come in many different forms, among which the missing bars defect has the greatest influence on lattice's behavior [4,5]. A number of researches have been carried out on mechanical properties of lattices with missing bars defect [4–6]. Hexagonal honeycomb is extremely sensitive to the presence of missing bars which causes a substantial knock down in the bulk modulus due to the induced bending stiffness of cell edge. It is found that with only 2% of missing cell edges a honeycomb's bulk modulus will be reduced to one hundredth of that of a perfect one. Kagome lattice is also sensitive to missing bars defect but the effect is not as strong as honeycomb while triangular lattice is not sensitive at all. The study [7] on the Kagome and triangular lattices single missing bar shows that the reduction in strength of Kagome lattice is greater than that of triangular lattice. Similar phenomenon is observed in 3D open cell foams that both Young's modulus and bulk modulus are reduced by the presence of broken cell edges [8]. Other mechanical properties [6,9,10] such as platform stress and energy absorption of lattices have also been investigated and the results reveal that all properties are degraded at different levels due to missing bars defect.

Obviously there is a pressing need to find a method to strengthen lattices' resistance to missing bars defect since lattices show great sensitivity to this imperfection. Latest researches [11–14] suggest that hierarchical structures have excellent mechanical properties. Study on the mechanical properties of mature honeycomb [15] shows that mature honeycomb is stronger and stiffer than newborn honeycomb as the silk cocoons which cover the surfaces of cell edges grow thicker. Similarly, by attaching low-density foam material to the shell surface of a cylindrical shell shows an improvement in its yield strength [16–19]. There were researchers building lattice material with sandwich structure cell edges and it turned out that both lattice's mechanical properties and imperfection sensitivity were upgraded [20]. In general, hierarchical materials have the potential to be a class of promising anti-defects materials.

Inspired by the researches on hierarchical structures, this paper builds hierarchical lattice materials by adding soft adhesion to cell edges on the basis of hexagonal, Kagome and triangular lattices. Finite element models of imperfect lattices are established and emphasis is placed on numerically investigating the influence of adhesion on lattices' imperfection sensitivity and deformation modes.

Fig. 1 shows three types of hierarchical lattices and the section of cell edge with adhesion. Three section parameters are defined as

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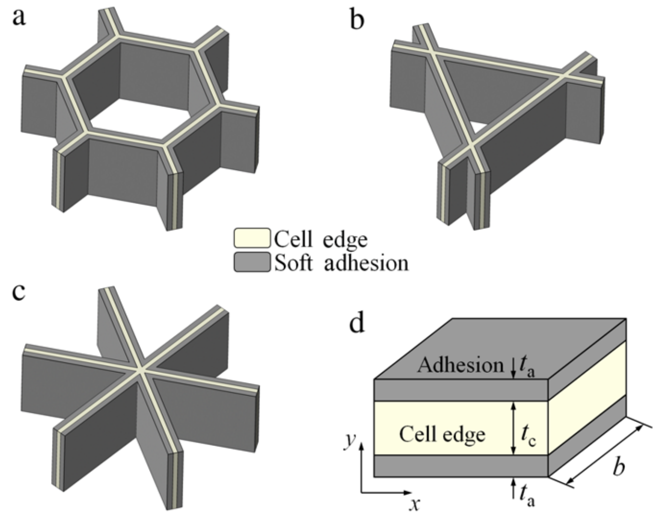

Fig. 1. Hierarchical lattices and cell edge with adhesion. (a) Hierarchical hexagonal lattice. (b) Hierarchical Kagome lattice. (c) Hierarchical triangular lattice. (d) Section of hierarchical lattice's cell edge.

follows in order to simplify the following contents:

$$\frac{t_a}{t_c} = k_t, \qquad \frac{E_a}{E_c} = k_E < 1, \qquad \frac{\rho_a}{\rho_c} = k_\rho < 1, \tag{1}$$

where t_a, E_a and ρ_a are respectively the thickness, Young's modulus and density of adhesion layer, t_c, E_c and ρ_c are respectively the thickness, Young's modulus and density of cell edge, as shown in Fig. 1(d). Meanwhile, the relative density ρ of hierarchical lattices is defined as

$$\rho = \frac{V_c\rho_c + V_a\rho_a}{V\rho_c}, \tag{2}$$

where ρ_c and ρ_a are the density of cell edge and adhesion layer, respectively, V_c and V_a are the corresponding volume. V is the overall volume of lattice.

Since the adhesion is much softer than lattice's parent material, the equivalent stiffness of hierarchical lattice's cell edge can be calculated on the basis of the Euler–Bernoulli beam theory. Hence the equivalent compressive stiffness $(AE)^*$ and equivalent bending stiffness $(EI)^*$ of cell edges with adhesion can be expressed as

$$(AE)^* = A_cE_c + A_aE_a = t_cbE_c + 2t_abE_a, \tag{3}$$

$$(EI)^* = \left(bt_c^3/12\right)\cdot E_c + \left(bt_a^3/6\right)\cdot E_a + \left[(t_c+t_a)^2\, bt_a/2\right]\cdot E_a, \tag{4}$$

where A_a and A_c are the section areas of adhesion layer and cell edge, while b is the out-plane thickness of lattice.

The ABAQUS/Standard version [21] was used here to establish the finite element models of imperfect hierarchical lattices, where a B22 Timoshenko beam element, which can be subjected to stretch, bend and shear, was used to represent each cell edge [4,22]. Meshes of the imperfect hierarchical lattice were generated from perfect parent meshes using a MATLAB routine [23], which randomly removed a proportion f of the elements. Note that f can take values of the range from 0 to 0.1. For a given level of imperfection f, the in-plane moduli were obtained by calculating the average bulk and shear moduli of twenty stochastic models with the same adhesion. The size of the lattice model was 48 × 48 unit cells which is large enough for the accuracy according to the study by Symons and Fleck [5]. The displacements of lattices' boundary nodes satisfy the periodic boundary condition expressed as follows [4,5,22]

$$\begin{cases} u_\alpha^J - u_\alpha^I = \varepsilon_{\alpha\beta}\left(x_\beta^J - x_\beta^I\right), \\ \theta^J - \theta^I = 0, \quad \alpha, \beta = 1, 2, \end{cases} \tag{5}$$

where $\varepsilon_{\alpha\beta}$ is the representative volume element's average micro strain, x_β^J and x_β^I are the coordinate values of the boundary nodes J and I before deformation, u_α^J and u_α^I are the linear displacements of J and I after deformation, and θ^J and θ^I are the angular displacements of J and I after deformation.

Since the adhesion layer is softer than the cell edge, the values of parameters k_t, k_E and k_ρ shall be carefully decided. According to the former study [24], six kinds of adhesion layers with different values of k_t and k_E were performed to investigate the influence of adhesion's mechanical and section properties on lattice's in-plane moduli. Specifically, k_t took values of 1 and 2 and k_E took values of 0.1, 0.01 and 0.001 while the value of k_ρ remained unchanged at 0.1. Notice that for each type of lattices their masses were constants regardless of the values of k_t and k_E as the relative densities k_ρ were unchanged, which gave the precondition for the comparison between the hierarchical lattice and their corresponding classic lattice.

The value of t/l of perfect classic lattice was taken as 0.02 [5] which indicated that the relative densities of hexagonal lattice, Kagome lattice and triangular lattice are respectively 0.023, 0.035 and 0.069, where t and l were the thickness and length of perfect lattice's cell edge, respectively. Numerical results of three types of hierarchical lattices with different adhesions under hydrostatic compression and shear loads are given as shown in Fig. 2, where f was lattices' defect degree, K and G were the imperfect lattice's bulk modulus and shear modulus respectively while K_0 and G_0 were the perfect lattice's bulk modulus and shear modulus respectively.

According to Fig. 2, hexagonal lattice's bulk modulus and Kagome lattice's bulk and shear moduli are sensitive to missing bars defect while hexagonal lattice's shear modulus and triangular lattice's bulk and shear moduli are insensitive. For those moduli which are imperfection sensitive, adding adhesion whose Young's modulus are one tenth of that of cell edges makes significantly improvement on the moduli. However when adhesion's Young's modulus are degraded the improvement turned into reduction. For those moduli which are imperfection insensitive, adding adhesion makes no differences whatever the adhesion's properties are.

To analyze how adhesion influences hierarchical lattice's imperfection sensitivity, the local deformation figures of lattices with a single edge missing are listed in Table 1. Note that all comparisons are based on the premise that lattices have the same relative density. It can be seen that the missing bars imperfection leads to lattice's deformation mode switches from stretching dominated to bending dominated except for hexagonal lattice under shear load [5]. Hexagonal lattice under hydrostatic compression load is most affected, followed in order by Kagome lattice and triangular lattice under hydrostatic compression and shear loads. According to Eq. (4), by adding adhesion of properties of $0.1\rho_c - 0.100E_c - 2t_c$ the bending stiffness of hierarchical lattice's cell edge is 4.88 times of that for the classic lattice's cell edge, which indicates that the bending deformation caused by missing bars defect is weakened and thus the lattice becomes insensitive to the imperfection. In contrast, when the adhesion's properties are $0.1\rho_c - 0.001E_c - 2t_c$ then the bending stiffness of hierarchical lattice's cell edge becomes 0.41 times of that for the classic lattice's cell edge, which indicates that the bending deformation is strengthened and the lattice becomes more sensitive to imperfection. In general, hierarchical lattice's imperfection sensitivity is reduced if the adhesion upgrades the bending stiffness of cell edge while adhesion degrading the bending stiffness does the opposite.

To measure the influence of missing bars defect on hierarchical and classic lattices, statistical analysis was conducted for the normalized node displacement of classic lattices and hierarchical lattices with adhesion of properties of $0.1E_c - 0.1\rho_c - 2t_c$, as shown