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Entropy-based adaptive attitude estimation

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ABSTRACT

Gaussian approximation filters have increasingly been developed to enhance the accuracy of attitude estimation in space missions. The effective employment of these algorithms demands accurate knowledge of system dynamics and measurement models, as well as their noise characteristics, which are usually unavailable or unreliable. An innovation-based adaptive filtering approach has been adopted as a solution to this problem; however, it exhibits two major challenges, namely appropriate window size selection and guaranteed assurance of positive definiteness for the estimated noise covariance matrices. The current work presents two novel techniques based on relative entropy and confidence level concepts in order to address the abovementioned drawbacks. The proposed adaptation techniques are applied to two nonlinear state estimation algorithms of the extended Kalman filter and cubature Kalman filter for attitude estimation of a low earth orbit satellite equipped with three-axis magnetometers and Sun sensors. The effectiveness of the proposed adaptation scheme is demonstrated by means of comprehensive sensitivity analysis on the system and environmental parameters by using extensive independent Monte Carlo simulations.

1. Introduction

Nonlinear state estimation methods within the Bayesian framework have increasingly been utilized in space navigation in order to enhance mission performance. The attitude determination (AD) subsystem plays a crucial role in most space navigation systems for achieving the desired goals. Although a considerable amount of AD-related research using the Kalman filter (KF) family exists, its implementation for satisfactory estimation results requires accurate prior knowledge of the measurement and process noise characteristics. Unfortunately, various system and environmental uncertainties cause the noise statistical properties to change over time; hence, in general, no exact prior information regarding noise parameters is available. The lack of sufficient a priori statistical noise characteristics results in filter performance degradation, while incorrect a priori information is a key cause of KF algorithm divergence. This divergence may also originate from other sources, such as irrelevant transition information and/or incorrect mathematical models and constraints. However, the primary focus of this study is to resolve the former problem. Although numerous diverse research schemes have been proposed to address this difficulty, all existing methods can be classified into four major categories, as follows.

1 $H\infty$ filtering: The $H\infty$ filter, also known as the minimax filter, makes no assumptions about noise statistics and attempts to minimize the worst-case estimation error. This estimation algorithm is a robust version of the KF method; however, it is over-conservative and offers lower estimation accuracy than the KF [1,2].
2 Adaptive Gaussian approximation filters: the adaptive-filtering strategy aims to tune the filter parameters, including noise statistics, based on variable working conditions. Therefore, lack of a priori noise statistical properties is compensated for, and the dependency of filter performance on noise parameters is reduced. These algorithms change the measurement and/or process noise characteristics by means of statistical investigation of the measurement residual or innovation. For this reason, such techniques are usually known as innovation-based adaptive estimation (IAE) methods. Kailath [3] was the first to propose the innovation sequence utility for KF tuning in 1968. IAE techniques differ in terms of the process frame and can be classified into several groups: covariance matching methods [4,5], maximum likelihood (ML) criterion [6], maximum a posteriori (MAP) criterion [7,8], autocorrelation of innovation, residual whitening, consistency tests [1,4], and master-slave structures [9,10]. The covariance matching approach has been considered as the fundamental concept underlying the majority of IAE methods.

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3 Multiple model adaptive estimation (MMAE): this method runs a bank of parallel KFs that differ in the utilized model. The MMAE method has been demonstrated as potentially viable for estimating every unknown parameter, such as noise covariance or dynamic/measurement system parameters. Despite its considerable computational effort and long execution time, this approach yields desirable estimation accuracy if the number of utilized models adequately covers the entire unknown parameter workspace; otherwise, it would fail and/or be relatively inefficient [11,12].
4 Artificial intelligence-based methods: these algorithms tune the noise strength based on heuristic methods originating from human experience, such as fuzzy logic [13], or inspired by natural phenomena, such as particle swarm optimization (PSO) [14], the ant colony (AC) algorithm [15], the genetic algorithm (GA) [16], and the tabu continuous AC system (TCACS) [17].

Although the IAE strategy offers simpler computation, a shorter run time, and a faster convergence rate, it suffers from two significant difficulties that limit its application in reality. Firstly, it cannot guarantee positive definiteness of the calculated noise covariance matrices during the numerical computer-based propagation process. Furthermore, noise estimation depends on the assumed window size for the undertaken innovation sequence. The window size has usually been adjusted in advance via ad-hoc strategies in the literature [18–20]. When considering time-dependent variations of noise properties and the dynamic system environment, online adjustment of window size is still important. It is for the latter part that this work proposes certain new ideas based on Cholesky decomposition, relative entropy, namely Kullback-Leibler distance (KLD), and the confidence level concept, in order to adapt the window size over time. The key contribution of this work is the creation of an appropriate working context for real implementation of the IAE method. Considering the nonlinear nature of the attitude estimation problem, the proposed adaptation scheme is implemented using two nonlinear filters, namely the extended Kalman filter (EKF) and cubature Kalman filter (CKF).

The remaining sections of this paper are arranged as follows. Satellite rotational motion, including attitude kinematics and dynamics, is modeled in section 2, and the measurement system is introduced in section 3. Section 4 is devoted to the development of adaptive entropy-based nonlinear filtering methods. Section 5 provides numerical simulations and a comprehensive sensitivity analysis of the various dynamic system parameters, adaptive estimation algorithm, and surrounding environment. Conclusions and future recommendations are provided in section 6.

2. Satellite rotational motion

The rotational motion of a satellite is described using attitude kinematics and dynamics. Various means exist for describing spacecraft attitude, including Euler angles, quaternion parameters, the Gibbs vector, and the direction cosine matrix [21]. Quaternion parameters are the most extensively used and preferred means of attitude representation because of their linear propagation equations and non-singular characteristics for any arbitrary rotation angle. As only three independent parameters are required for attitude representation, the unit norm constraint seems as a disadvantage, since any extra constraint imposes additional computational effort and complexity. The attitude represented by the quaternion parameters $\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ can propagate in time, as follows [22]:

$$\frac{d}{dt}\mathbf{q} = \frac{1}{2}\begin{bmatrix} K\varepsilon & \omega_z & -\omega_y & \omega_x \\ -\omega_z & K\varepsilon & \omega_x & \omega_y \\ \omega_y & -\omega_x & K\varepsilon & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & K\varepsilon \end{bmatrix}\mathbf{q}, \tag{1}$$

where $\varepsilon = 1 - \mathbf{q}^T\mathbf{q}$ and $q_4$is considered as the scalar part of $\mathbf{q}$. The diagonal matrix elements are specifically selected as presented in Eq. (1) in order to guarantee the unit norm condition of quaternions, even in the presence of rounding errors. Moreover, K is an arbitrary constant selected so that $K\Delta t \le 1$, where Δt is the integration time step, while $\boldsymbol{\omega}_{BI} = [\omega_x \quad \omega_y \quad \omega_z]^T$ represents the angular velocity vector of the spacecraft body (B) with respect to the reference inertial frame (I), expressed in the body frame (B).

The nonlinear attitude dynamics of a rigid satellite can be described by Euler's law. Expressing Euler's law in the satellite body coordinate system results in Ref. [23]:

$$[\dot{\boldsymbol{\omega}}_{BI}]^B = I_B^{B-1}\left([\boldsymbol{\tau}]^B - [\boldsymbol{\omega}_{BI}]^B \times I_B^B[\boldsymbol{\omega}_{BI}]^B\right), \tag{2}$$

where $[.]^B$ denotes the quantity expressed in the body coordinate system, I_B^B is the spacecraft moment of inertia (MOI) matrix about its center of mass, expressed in the body frame, and $\boldsymbol{\tau}$ is the total torque exerted on the spacecraft, including the control and disturbance. Disturbances applied to an earth satellite originate from various internal and/or external sources, such as aerodynamic drag, solar radiation pressure, gravity gradient, electromagnetic torque, and fuel sloshing, while the severity of most disturbing forces/moments depends on the space vehicle altitude. As the aerodynamic drag, gravity gradient, and residual magnetic moment are the most effective disturbance torque sources for a small satellite in LEO [23], these factors are considered in the present study.

3. Measurement system

The measurement system is usually selected according to mission accuracy, AD requirements, and project budget. In order to achieve a low-cost but precise navigation system, a centralized fusion of the micro-electro-mechanical system (MEMS)-based three-axis magnetometer (TAM) and the Sun sensor are selected to meet the performance and budget requirements simultaneously.

The low cost, light weight, and low power requirements offered by TAMs have made them useful sensors in most LEO missions [24]. TAMs measure the geomagnetic field in the satellite body coordinate system; therefore, the TAM output can be modeled as:

$$[\mathbf{B}_{meas}]^B = T^{BI}[\mathbf{B}_{\text{model}}]^I + \mathbf{v}_B, \tag{3}$$

where $\mathbf{B}_{\text{meas}}$and $\mathbf{B}_{\text{model}}$represent the measured geomagnetic field vector and its corresponding theoretical counterpart obtained from existing models, respectively. Furthermore, $\mathbf{B}_{\text{model}}$ is a function of the satellite position, and is extracted from the international geomagnetic reference field (IGRF) [25] in the current work; $\mathbf{v}_B$represents the corresponding measurement noise, assumed as zero-mean Gaussian with a variance of σ_B^2 along each axis; and T^{BI} represents the transformation matrix of the inertial frame to the body frame, defined in terms of the quaternion parameters, as follows [22]:

$$T^{BI} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}, \tag{4}$$

The Sun sensor is the other utilized reference sensor that provides sunlight direction with respect to the sensor frame assumed to be coincident to the satellite body frame. The sensor output in the satellite body coordinate system is modeled as follows:

$$[\mathbf{s}_{meas}]^B = T^{BI}[\mathbf{s}_{\text{ref}}]^I + \mathbf{v}_s. \tag{5}$$

where $\mathbf{s}_{meas}$ and $\mathbf{s}_{\text{ref}}$ are the measured and modeled Sun direction vectors, respectively, and $\mathbf{s}_{\text{ref}}$ can easily be obtained as the difference between the Sun and satellite position vectors; and $\mathbf{v}_s$ is the Sun sensor measurement noise, modeled as zero-mean Gaussian white noise with a variance of σ_s^2 along each axis. Moreover, no noise correlation is assumed between the