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Influence of the thermo-electric coupling on the heat transfer in cylindrical annulus with a dielectric fluid under microgravity

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ABSTRACT

The present note gives the result of direct numerical simulations of the convective flow induced by the dielectric force in a cylindrical annulus under microgravity conditions. A dielectric fluid confined between two coaxial cylinders is subject to a high-frequency tension and a radial temperature gradient. The resulting buoyancy force creates a supercritical convective flow when the critical value of the electric Rayleigh number is exceeded. This flow is made of stationary helicoidal vortices. The effect of the thermo-electric coupling is sensitive for large values of the radius ratio: it stabilizes the conductive state and reduces the slope of the increase of the heat transfer and of the kinetic energy.

1. Introduction

Flows induced by the artificial radial gravity play a significant role in the modeling geophysical and astrophysical flows and in the control of the heat transfer. In fact, because of the locally oriented vertical gravity, it is quite to difficult to realize experiments that mimic accurately the geophysical flows in Earth environment. A temperature gradient can be also applied in order to create a density stratification and therefore to generate baroclinic flows.

The modeling of geophysical or astrophysical flows has been made by the flows in cylindrical annulus in solid rotation around the axis and with a radial temperature gradient [1,2]. The centrifugal acceleration induced by the solid body rotation reproduces the gravity only in the thin layer of the fluid while the real gravity varies in the radial direction. The electric gravity induced by the dielectrophoretic force decreases with the radial distance as r^{-5} for spherical gaps [3–6] and as r^{-3} [7,8] for cylindrical annulus. That this the reason why the dielectrophoretic force has proved to be a good candidate for modeling of thermal convection in planets and in stars [9,10]. Our investigation is motivated by the recent experiment GEOFLOW realized on ISS with thermo-electric convection induced by dielectrophoretic force in a spherical gap [10]; the purpose of the present study is to analyze in detail the structure of the convective flow induced by the thermo-electric convection in the case of cylindrical annulus.

We focus our attention on the production of the artificial gravity by applying of the fast oscillated electric field between cylindrical surfaces (Fig. 1) containing a fluid of density ρ and permittivity ϵ. The electric gravity occurs because of the dielectrophoretic effect related to the coupling between the electric field $\mathbf{E}$ and the gradient of the fluid permittivity. The dielectrophoretic force density is given by [11]

$$\mathbf{f}_{dep} = -\frac{1}{2}E^2\nabla\epsilon \tag{1}$$

The dielectrophoretic force is the main acting force as far as the frequency of the electric field f is much larger than the inverse of the charge relaxation time $\tau_e^{-1} = \sigma/\epsilon$ (where σ is the electrical conductivity) and of the fluid characteristic times. The dielectrophoretic force contains a conservative part and a buoyancy force that can be written as follows

$$\mathbf{f}_{dep} = -\rho_0\alpha(T - T_2)\mathbf{g}_e + \nabla\left(\frac{e\epsilon_2(T - T_2)E^2}{2}\right) \quad \mathbf{g}_e = \frac{e}{\alpha\rho_0}\nabla\frac{\epsilon_2 E^2}{2}, \tag{2}$$

where the temperature dependence of the density and the dielectric constant of the fluid can be approximated by linear functions

$$\rho(T) = \rho_0[1 - \alpha(T - T_2)], \quad \rho_0 = \rho(T_2), \quad \epsilon(T) = \epsilon_2[1 - e(T - T_2)], \quad \epsilon_2 = \epsilon(T_2). \tag{3}$$

α and e are the thermal expansion and the thermal coefficient of the permittivity. So the variation of the fluid permittivity ϵ with the temperature dependence in the inhomogeneous field

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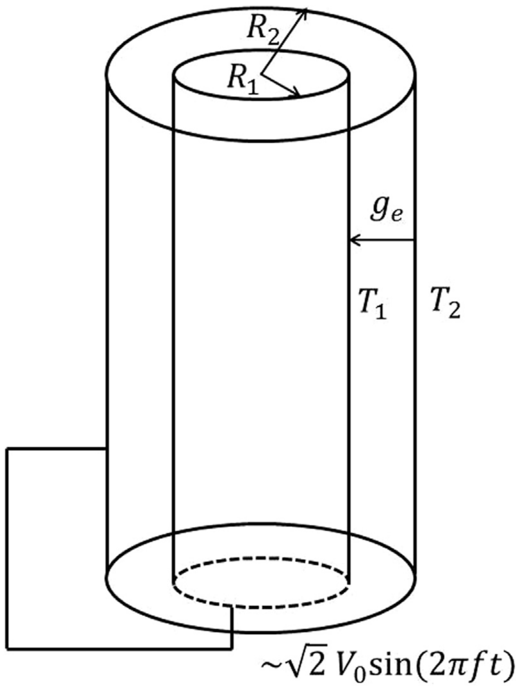

Fig. 1. Sketch of the cylindrical annulus with an applied alternating electric tension and a radial temperature gradient.

generates an electric gravity occurs $\mathbf{g}_e$ so that the dielectric force is associated with the buoyancy and therefore it can generate thermal convection in analogy with the classical Rayleigh-Bénard (RB) convection between two planes. While in the RB convection, the gravity is constant, the electric gravity in the cylindrical annulus depends on the radial coordinate as $g_e \sim r^{-3}$. For relatively small values of the temperature gradients, the basic state is a conductive regime with $\mathbf{u}_0 = 0$. In the case of infinitely long cylindrical annulus, the linear stability analysis [7] shows that the basic flow loses its stability to three-dimensionally steady perturbations that manifest themselves in form of helicoidal vortices. The critical electrical Rayleigh number, L_c, is independent on the Prandtl number and increases with increasing the radius ratio η. The critical wave number in the azimuthal direction, m_c, increases drastically with η. The occurring supercritical three-dimensional flow can influence an applied electric field, too. The corresponding effect called *thermo-electric coupling* becomes significant according to the linear stability analysis for $\eta > 0.6$. We investigate the influence of the thermo-electric coupling effect under the microgravity on the heat transfer and amplitudes of the supercritical flows. It is an important sequel of the previous work [8] that was realized for small values of the radius ratio for which the thermo-electric coupling effect is negligible (Table 1).

2. Equations

In the present work, we will assume the validity of the Boussinesq approximation for the fluid density and permittivity, i.e. the temperature difference between two cylindrical surfaces $\Delta T = T_1 - T_2 > 0$ is assumed to be sufficiently small and the viscosity and thermal diffusivity are constant. The following scaling has been used to get the equations in the dimensionless form: $r = r^*d$, $t = t^*d^2/k$, $u = u^*k/d$, $T - T_2 = \Delta T\Theta$, $\Phi = \Phi^* V_0$ where d is the gap width of the cylindrical annulus and V_0 is the potential applied to the inner electrode. The Navier-Stokes equation, the energy equation, the continuity and the Gauss equation must be solved together to find the velocity field and the temperature. They read

Table 1
List of parameters.

$\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$	unit vectors in radial, azimuthal and axial directions
r	radial coordinate
x	radial coordinate ($x \in [-1, 1]$), $r(x) = \frac{1}{2}\left[x + \frac{1+\eta}{1-\eta}\right]$
$\mathbf{u}$	velocity field
u_r, u_θ, u_z	radial, polar and azimuthal velocity components
p	pressure
$\Phi_b(r)$, $\Theta_b(r)$	electric potential, temperature (basic state)
$\Phi(r,\theta,z)$, $\Theta(r,\theta,z)$	electric potential, temperature (3D flow)
$\tilde{\Phi}(r,\theta,z)$	electric potential ($\Phi(r,\theta,z) - \Phi_b(r)$)
t, Δt	time, time step
R_1, R_2, d	inner radius, outer radius, $d = R_2 - R_1$
L	electric Rayleigh number, $\alpha g\Delta T d^3/\nu k$
Pr	Prandtl number, ν/k
ΔT	$T_1 - T_2$ temperature difference
m_c	critical azimuthal wave number
κ_c	critical axial wave number
Greek symbols	
α	volume expansion constant
ν	kinematic viscosity
η	radius ratio, R_1/R_2
k	thermal diffusivity
ρ	density
ϵ	permittivity
e	thermal coefficient of permittivity
γ_e	$e\Delta T$

where we have omitted the stars on dimensionless quantities

$$Pr^{-1}\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = -\nabla P + \Delta\mathbf{u} - LT\mathbf{g}_e \tag{4}$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u}\cdot\nabla\Theta = \Delta\Theta \tag{5}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{6}$$

$$\nabla\cdot(\epsilon\mathbf{E}) = 0, \quad \mathbf{E} = -\nabla\Phi \tag{7}$$

where

$$L = \frac{\alpha g_0 \Delta T D^3}{\nu k} \tag{8}$$

is the electric Rayleigh number and the electric gravity g_0 is calculated at the middle of the gap $r_0 = (1+\eta)/2(1-\eta)$.

The velocity satisfies the no slip boundary conditions i.e $\mathbf{u}=0$, the temperature field and the potential obey the following conditions

$$\Theta = 1, \Phi = 1, \quad \Theta = 0, \Phi = 0 \tag{9}$$

at $r = \eta/(1-\eta)$ and $r = 1/(1-\eta)$, respectively. The equations Eqs. (4)–(9) must be solved in the domain $[r \in \eta/(1-\eta), 1/(1-\eta)] \times [\theta \in 0, 2\pi] \times [z \in 0, \lambda_c]$. The periodic structure of the flow and the temperature in the axial direction can be presented in terms of the wave length λ_c. This value can be derived by means of the linear stability analysis.

For nonlinear analysis of the thermo-electric coupling effect, the electric potential Φ can be represented as a sum of two terms