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Position determination and measurement error analysis for the spherical proof mass with optical shadow sensing

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ABSTRACT

To meet the very demanding requirements for space gravity detection, the gravitational reference sensor (GRS) as the key payload needs to offer the relative position of the proof mass with extraordinarily high precision and low disturbance. The position determination and error analysis for the GRS with a spherical proof mass is addressed. Firstly the concept of measuring the freely falling proof mass with optical shadow sensors is presented. Then, based on the optical signal model, the general formula for position determination is derived. Two types of measurement system are proposed, for which the analytical solution to the three-dimensional position can be attained. Thirdly, with the assumption of Gaussian beams, the error propagation models for the variation of spot size and optical power, the effect of beam divergence, the chattering of beam center, and the deviation of beam direction are given respectively. Finally, the numerical simulations taken into account of the model uncertainty of beam divergence, spherical edge and beam diffraction are carried out to validate the performance of the error propagation models. The results show that these models can be used to estimate the effect of error source with an acceptable accuracy which is better than 20%. Moreover, the simulation for the three-dimensional position determination with one of the proposed measurement system shows that the position error is just comparable to the error of the output of each sensor.

1. Introduction

Space gravity detection, aimed to discover the specific gravity signal with extremely weak magnitude or measure the modest gravity signal with unprecedented precision, has ever been an interested research scope for scientists devoted to astronautics [1], geodesy [2], physics [3], precision instruments [4], etc. The successfully launched CHAMP [5], GRACE [6], GOCE [7] are milestones in the Earth's gravity measurement. The involved inertial measurement and drag-free control technology have promoted the development of the sun centered orbit mission LISA [8], which is proposed to detect the gravitational wave. Other relevant missions include the GP-B for the test of general relativity [9], the STEP intended to test the equivalence principle [10], and the inner formation flying system (IFFS) of which first application is to recover the Earth's gravity field [11].

While space gravity detection has so much prospects and significance, substantial challenges are confronted as well. One of them is to construct the gravitational reference sensor (GRS). The GRS is typically composed of a proof mass and the cavity that contains it. It can be divided into two categories generally, one is also called as accelerometers which measure the non-gravitational disturbance exerted on the satellite, and the other is designed to output the relative position of the proof mass with respect to the cavity. For the latter, the proof mass used as a purely gravity reference point, should fall freely at least along one translational degree of freedom within the cavity. For instance, each cubic proof mass of the LISA spacecraft falls freely along the sensitive direction, and the other degrees of freedom are maintained by electrostatic suspension force [12]. In contrast, the spherical proof mass for the IFFS flies along a purely gravitational orbit without any applied forces [13].

Inspired by the concept of satellite formation flying, the proof mass within the GRS of the IFFS is also named as inner satellite, and the satellite providing the cavity is called as outer satellite. Therefore, the inner and outer satellite can form a special satellite formation defined as inner formation [14]. A satellite system to recover the Earth's gravity field based on the IFFS is conceived. In contrast to the CHAMP employing accelerometers to measure the non-gravitational forces applied on the satellite, the IFFS based satellite system utilizes the precise orbit data of the inner satellite to measure the Earth's gravity field. The GRS onboard the IFFS

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features large cavity gap which benefits the further improvement of the purely gravitational environment. Besides that, smaller coupling stiffness between the proof mass and the outer satellite is anticipated as its major contribution is self-gravity gradient which decreases rapidly by enlarging the cavity gap [15]. Hence, the requirement for the relative motion amplitude can be relaxed. Perfect disturbance compensation is not essential, and precise formation flying control technology can be developed to track the proof mass using thrusters [16,17].

For the IFFS-type GRS, the relative sensing technology that accommodates to large cavity gap is required. The mission matched measurement precision and dynamic range, as well as the introduced non-gravitational disturbance on the proof mass, are major considerations for the relative measurement design. For the IFFS, an infrared imaging system installed on the cavity with a radius of 215 mm is designed to reach a precision lower than 0.1 mm which meets the requirement to recover the Earth's gravity field [18]. As a completely passive method, a prominent advantage of infrared imaging is that no non-gravitational force is generated during the measurement process. Despite the infrared imaging system performs well, another measurement technology that can reach a precision of about 1 μm or 1 nm is required for the composite inner formation flying system (CIFFS) which is composed of two IFFSs along one orbit. Similar to the GRACE, the CIFFS uses the relative distance between two remote proof masses to recover the Earth's gravity field. To match the performance of the microwave ranging system employed for the distance measurement between two outer satellites, a precision of 1 μm is needed. Alternatively, if a laser interferometry system is adopted, then a precision of 1 nm is matched.

The capacitive sensing and optical sensing are two commonly used techniques for the proof mass measurement. Capacitive sensing has been widely employed for the construction of accelerometers [19]. While it can reach a precision of about 1 nm/Hz$^{1/2}$, the limitation to the cavity gap below several millimeters may be a problem for its application in an IFFS-type GRS [20]. Moreover, the disturbance induced by the fluctuation of the applied voltage of the electrodes may be considerable for advanced space gravity detection missions [21]. Generally, the optical sensing has the potential to achieve high precision and weak disturbance, and can be implemented without restrictions on the cavity gap [22]. For these reasons, Debra considers that the optical sensing should be the prior choice for the design of the future GRS [23].

As a promising measurement technology for the proof mass, the optical shadow sensing uses the optical power of the measuring beam partially blocked by the proof mass to obtain the displacement of the proof mass [24]. The optical shadow sensing is adopted by the modular gravitational reference sensor (MGRS) as a complementary method to the interferometric optical sensing [25]. The MGRS is conceived as an alternative to the GRS of the LISA, moreover a candidate for the future gravitational wave detection missions. One spherical proof mass freely falling inside the cavity is used for the MGRS, and the optical shadow sensing provides feedback to the drag-free control with a desired precision of several nanometers [26]. Except that the control requirement for the suppression of displacement related disturbance can be met, its wide range of 1 mm is vital to address the unexpected displacement caused by initial releasing or hitting by debris to the spacecraft, etc. [27]. The optical shadow sensing is also adopted by the CIFFS to reach at least a precision of 1μm and a dynamic range of 3 mm [28]. To maximize the responsivity of photodetector, near-infrared beams should be a prior choice. As the sensor back-action acceleration is a special and important factor for the space application, the optical power should be limited below about 1mW to reduce the optical radiation pressure to be no larger than 10^{-12}N [29]. Besides the space application, the optical shadow sensing can also be employed to measure the mass center of a spherical proof mass on ground [30], and develop the 'violin-mode' shadow sensor for the displacement measurement of the suspension mirror in the ground-based gravitational wave detection system LIGO and aLIGO [31,32]. As for the mentioned ground application, the arrangement of sensors is intuitive as only one-dimensional motion is sensed. However, for the three-dimensional measurement, multiple optical shadow sensors should be disposed in a reasonable way to perform well. For the MGRS, a system composed of four sensors is designed, with each pair of sensor emits parallel beams and the measured dimensions of the two pairs are perpendicular [26]. A general relationship between the sensor output and position of the proof mass would greatly promote the optimal design of multi-sensor system.

To make the optical shadow sensing suffice the demanding mission requirements, the various types of error source should be investigated to improve measurement precision. Thrittler has established a one-dimensional experimental setup to study the noise floor [27]. Long-term temperature tests are conducted to suggest that the distinctly large noise in the low frequency band below 1 Hz is mainly caused by the temperature dependent electrical drift. Thus, a lock-in amplifier is used to overcome this type of noise for the improved experiment [26]. Except for that, three main noise sources, i.e. electrical noise, beam jitter and laser amplitude noise are discussed in Ref. [27]. The electrical noise composed of shot and Johnson noise is calculated to be negligible, whereas the beam jitter observed by a CCD camera is deduced to cause a large error of about 0.1 mV with a rough estimation. The results of several experiments show that the secular fluctuation of laser amplitude is not ideally cancelled by the differential design, and the exact explanation should be further investigated. The fiber optics, integrated mechanical structure and frequency modulation are employed to suppress noise for the third generation of experiment [26]. Despite that some major error sources have been experimentally discussed, it is important to attain the quantitative error propagation relationships. They can contribute to establish the requirements for sensor parameters, e.g. beam variation and mechanical misalignment, to reach expected performance.

Herein the three-dimensional position determination and error propagation analysis for a spherical proof inside an IFFS-type GRS are conducted, and the organization is given as follows. Section 2 presents the concept of the relative measurement of a spherical proof mass with optical shadow sensors. Additionally, the optical signals with parallel and diverged beam are modeled respectively as the fundamental of the following study. In Section 3, the general formula to relate the three-dimensional position of the proof mass to the output of multiple optical shadow sensors is established. Based on the formula, two types of sensor system with specific configuration are designed. The proof mass can be positioned uniquely by using these two systems. For the case of Gaussian beams, the error propagation models for five major error sources are derived in Section 4. In Section 5, the error propagation models are validated through simulations in view of the effect of error coupling and uncertainty of signal model.

2. Concept of measuring a spherical proof mass with optical shadow sensors

The concept of measuring a spherical proof mass using optical shadow sensing is shown in Fig. 1. The light beam emitted from a light source is partially blocked by the proof mass nominally centered in the cavity, thus the remaining optical power detected by the opposite photodiode changes with the movement of the proof mass. Then the optical signal is converted by the photodiode into an analog signal which goes through an amplifier and AD