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Covariance study to evaluate the influence of optical follow-up strategies on estimated orbital parameters

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ABSTRACT

An in-depth study, using simulations and covariance analysis, is performed to identify the optimal sequence of observations to obtain the most accurate orbit propagation. The accuracy of the results of an orbit determination/improvement process depends on: tracklet length, number of observations, type of orbit, astrometric error, time interval between tracklets and observation geometry. The latter depends on the position of the object along its orbit and the location of the observing station. This covariance analysis aims to optimize the observation strategy taking into account the influence of the orbit shape, of the relative object-observer geometry and the interval between observations.

1. Introduction

At the moment there are more than 29,000 objects with a diameter bigger than 10 cm, and more than 670,000 objects with a diameter bigger than 1 cm in the space around the Earth; furthermore, according to estimates there are more than 170 million objects bigger than 1 mm [1]. Among all these objects, only about 1400 are active satellites, all the rest is space debris. Space debris is any man made orbiting object which is not operational with no reasonable expectation of assuming or resuming its intended function [2]. The space debris population includes discarded satellites, rocket bodies, mission-related objects, painting and insulation flakes, fragments created by collisions, and break up events [3].

Space debris constitutes a serious problem for space missions, both humans and satellites. In fact, in the Low Earth Orbit region (LEO) for example, the average speed for debris is about 8 km per second [4]; while in the Geostationary region (GEO) the average speed for debris is more than 3 km. Because of the high velocities of the debris particles, the present shields, including those used on-board of the International Space Station (ISS), are able to protect spacecraft only from the smaller debris (less than 1 cm in size [3]). For these reasons the space debris is an important topic for the various space agencies and institutions, which are conducting a lot of research to better understand this problem. The Astronomical Institute of the University of Bern (AIUB) is also involved in this field of research. The most common unanswered questions are: how many debris objects are there? What are the most populated regions? What are they made of? And how will this population evolve in the future? To answer these questions the most common approach consists of three main steps: the first is the discovery of the objects [5], the second is the orbit determination [6,7] and the third is the characterization of the objects [8,9]. The AIUB performs all these activities using its telescopes. As mentioned before, the first orbit determination and the orbit improvement are fundamental steps for the study of space debris. The first one is performed by scanning certain regions of the sky chosen in a way to ensure that an object is observed several times during the same night [10]. The second is performed by planning regular observations of the object of interest, these additional series of observations are usually called follow-ups. Due to the huge amount of space

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debris and to the limitations of the telescopes, which can operate only when the weather conditions are good, it is necessary to optimize the time available for follow-up observations.

This paper will describe a method, based on the analysis of the covariance matrix, to understand how the follow-up observations should be distributed to obtain the best orbit; in addition this method can also be applied to optimize the survey strategies. This method will not provide a general rule on how to distribute observations but the idea is to suggest which observation strategy adopt having an a priori knowledge of the orbit. Thus, in the case of follow-up one already knows, even roughly, the orbit of the interested object and the method will tell where to observe to improve certain parameters (according to the user needs). In the survey case, the user already knows which kind of orbital class wants to investigate, thus the proposed method will provide information about which orbital regions to observe in order to obtain the best initial orbit. In this paper we will focus mainly on high altitude orbital regions because, especially the GEO regions, they have a high concentration of space debris and can be observed mainly using optical sensors. We define the best orbit as the set of orbital elements with the lowest uncertainties, that maximizes the accuracy of the predicted positions of the object. In the first part of the paper we describe the reasons that brought us to use the covariance matrix for this study and we will show the results of a theoretical study carried out to identify the main parameters which influence the results and how the problem can be simplified. Then the results obtained from a simplified scenario will be shown. Afterwards the complexity of the scenario is increased step by step showing in details the consequences on the results. Finally, this paper will present the results obtained from the application of the covariance study to some typical observation scenarios.

2. Theory

At the end of an observation night is not unusual to have an average of two tracklets per observed objects. A tracklet is the result of a series of images acquired during a survey campaign or during a follow-up of an already cataloged object. We assume that a standard tracklet is consisting of e.g. 7 images, each one of them contains a triplet of measurements, two angular, one in Right Ascension and one in Declination (respectively RA and DE), and a time epoch. This means that on average, from an observations night, one has 28 angular observations and 14 epochs for each object. Of course, these numbers can vary depending on factors as: the number of objects to observe in the catalog, the survey strategy and also, the performance of the software used to extract the measurements from the images. For this study we assumed to have two good tracklets per observed object. These two series of observations are then used to determine/improve the orbit of the object by mean of a Least SQuares adjustment (LSQ). The aim of this study is to analyze the output covariance of a LSQ process to understand how the geometry between observer and target object influences the accuracy of the estimated parameters. This analysis is carried out in order to be able to optimize the parameter estimation and to find the best combination of tracklets which gives the best orbit.

2.1. What is the covariance

The covariance matrix was chosen as the evaluation criterion because, as one can see from Eq. (1), it contains the uncertainties of the estimated parameters only as a function of the partial derivatives of the observations w.r.t. them; or rather it contains the partial derivatives of RA and DE w.r.t. semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee and the 6th parameter, see Eq. (2). These partial derivatives are functions of the geometric relation between observer and observed object.

$$P = m^2[A^T WA]^{-1}, \tag{1}$$

in which:

$$A = \frac{\mathrm{d}obs_i}{\mathrm{d}X_o}, \tag{2}$$

where P is the covariance matrix; m^2 is the a posteriori variance factor [11]; A is the first design matrix; W is the weight matrix; $obs_i = [RA_i; DE_i]$ are the ith angular measurements, respectively Right Ascension and Declination, where $i = 1,\ldots,n$ and n is the number of observations; $X_o = \left[a, e, i, \Omega, \omega, 6\text{th}\right]$ are the orbital parameters.

The a posteriori variance factor is function of the residuals of the observations in RA and DE. In reality the residuals are function of the astrometric error on the measurements and can be also due to deficiencies in the mathematical model used in the LSQ. This quantity is not taken into account in this study because our aim is not to solve the parameters estimation problem but is only to evaluate the influence of the object-observer relative geometry on their accuracy.

To carry out this study it was assumed that the observations are performed from the same telescope system and then we do not have to address the problem of measurements with different qualities [11,12]. This assumption allow us to make a further simplification regarding the weight matrix (W) that in this case is a unit matrix of size $2n$. Another important information is contained within the correlation indices which can be retrieved from the covariance matrix as shown in Eq. (3). These indices are useful because they tell us how strong any two parameters are correlated:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}, \tag{3}$$

where $-1 \le \rho \le 1$ is the correlation index; σ_{ij} is covariance of the elements i and j; σ_i and σ_j are the standard deviations of the elements i and j.

2.2. Choice of parameters

As can be seen from Eqs. (1) and (2), the output covariance of a LSQ adjustment is a $[6 \times 6]$ matrix function of