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Spacecraft dynamics under the action of Y-dot magnetic control law

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A B S T R A C T

The paper investigates the dynamic behavior of a spacecraft when a single magnetic torque-rod is used for achieving a pure spin condition by means of the so-called Y-dot control law. Global asymptotic convergence to a pure spin condition is proven on analytical grounds when the dipole moment is proportional to the rate of variation of the component of the magnetic field along the desired spin axis. Convergence of the spin axis towards the orbit normal is then explained by estimating the average magnetic control torque over one orbit. The validity of the analytical results, based on some simplifying assumptions and approximations, is finally investigated by means of numerical simulation for a fully non-linear attitude dynamic model, featuring a tilted dipole model for Earth's magnetic field. The analysis aims to support, in the framework of a sound mathematical basis, the development of effective control laws in realistic mission scenarios. Results are presented and discussed for relevant test cases.

1. Introduction

This paper deals with magnetic control of small spacecraft based on the so-called Y-dot control law. The case of spacecraft with arbitrary inertia tensor using a single magnetic torque-rod for detumbling is considered and the analysis is focused on the property of generalized exponential asymptotic stability in variations (GEASV) of the pure-spin equilibrium points. Also, the mechanism of the alignment of the spin axis to the orbit normal in a general geomagnetic field is explained in physical terms, and proved analytically. As a further contribution, a comprehensive numerical study is performed in order to evaluate the relationship between the value of the (uncontrolled) final pure-spin rate, on one side, and spacecraft initial rotational energy and inertia tensor on the other one.

In the framework of small satellite, magnetic control has been increasingly gaining in popularity. Ease of design, absence of moving parts, low mass, and need for renewable-only electrical power to operate make magnetic actuator quite appealing for space applications, especially for low Earth orbit missions where the magnitude of the magnetic field is stronger. As a drawback, magnetic-only controlled system are inherently underactuated, as no torque can be produced along the direction of the instantaneous local magnetic field vector. In this respect, practical control solutions exploit the variability of the geomagnetic field as sensed by the spacecraft, which primarily depends on spacecraft attitude dynamics and orbital motion. Magnetic control proved to be a viable solution for a variety of mission tasks, spanning from

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(coarse) full attitude stabilization [1,2], reaction or momentum bias wheels desaturation [3], to initial detumbling after spacecraft separation [4]. The latter application is considered in the present study.

The detumbling phase is crucial for small spacecraft, which are often launched as a secondary payload (aside a more important/expensive payload) and released at an unknown, and often high, tumbling rate. A reliable, rapid, and cost-effective reduction of the angular rate below a given threshold is commonly required prior to the deployment of solar arrays, when present, as well as any other mission-related task (such as Sun acquisition, initial attitude determination, Earth pointing, etc.).

Magnetic detumbling of a spacecraft may be attained by means of simple command laws such as the B-dot controller [4]. The B-dot command law allows one to drive the initial angular velocity to (almost) zero, without requiring attitude or rate information. Only measurements of the Earth magnetic field are required. Theoretical [5] and practical [6] aspects of this controller has been extensively studied, proving the viability of the approach and estimating its expected performance.

Several variants of the B-dot controller have been proposed over the last few years. Many of them aim to drive the spacecraft towards a non-zero angular velocity, where a final spinning condition allows one to reduce thermal loads, by avoiding prolonged exposition of the Sun-facing side to solar radiation. In other cases, a residual spinning motion is required to preserve a certain amount of angular momentum necessary to spin up the momentum wheel that provides gyroscopic stabilization [7].

In the present paper, a single-axis B-dot control law, also known as β-dot or Y-dot command law (due to the fact that the pitch, y, axis is the most common choice as commanded axis) is considered. By using a single magnetic torque-rod this controller (usually adopted in conjunction to a spin-rate controller, which makes use of an additional orthogonally placed magnetic torque-rod) can reduce the initial tumbling rate, eventually steering the satellite either towards a pure spin condition around one of its principal axes of inertia or to an almost zero residual angular rate. Its effectiveness was demonstrated through simulations before being successfully applied in several missions, such as SNAP-1 [8], FalconSAT-3 [9], UoSAT-12 [10], Alsat-1 [11], and SUMBANDILASAT [12].

Despite the conceptual simplicity of a Y-dot controller, a rigorous analysis of closed-loop system dynamics, characterizing the properties of the resulting motion (e.g., global asymptotic convergence towards a terminal pure-spin condition, spin-axis direction with respect to the orbit frame, etc.), is far from trivial. Nevertheless, such an analysis is necessary for supporting a mathematically sound approach to the synthesis of a control system which guarantees adequate performance in an realistic mission scenario.

This subject was recently investigated in [13], where the actions of both B-dot and Y-dot controllers were considered within a simplified, "circular", geomagnetic field model. Under the assumptions that (i) magnetic induction vector describes a circular cone in the orbit frame (rather than an elliptical one, as it results from the widely used tilted dipole model [14]), and (ii) the spacecraft is an axisymmetric rigid body, an averaging technique allowed for an elegant analytical derivation of relevant controller properties. A further numerical analysis, which retains the same simplifying assumptions on the geomagnetic field, was presented in [15].

The present paper aims at generalizing these results, by proving global asymptotic convergence to a pure spin state for the Y-dot control law even in the case of a tri-inertial spacecraft and without further assumptions on the geomagnetic field model. A variational system approach [16] to the stability analysis of non-linear systems is pursued. The equations of motion for system dynamics are recast in the form of a nominal system perturbed by a vanishing perturbation term. Global exponential asymptotic stability (EAS) of the origin for the nominal system is proven first. Robustness of the stability of the nominal system is then invoked in order to derive generalized exponential asymptotic stability in variations (GEASV) of the perturbed system [17]. The inertial position of the spin axis is also considered, proving that, under the assumptions valid for a tilted-dipole geomagnetic field model, the Y-dot control law eventually aligns the spin axis with the direction normal to the orbit plane, regardless of the initial condition. The convergence of the spin axis towards the orbit normal is proven by estimating the average magnetic control torque over one orbit.

The study is completed by a detailed numerical analysis of the spacecraft pure-spin final state as a function of the initial tumbling motion and spacecraft mass distribution, assuming that no control is performed other than the Y-dot (i.e., using only one magnetic torque-rod). In particular, the relationship between final pitch rate, initial spacecraft rotational energy, and spacecraft inertia matrix is analyzed, identifying those conditions that may lead to a complete detumbling of the spacecraft, which is quite a relevant feature for the practical application of this command law. An insightful explanation is proposed in terms of polhode curves and energy-dissipation. It is also proven that the adopted control law aligns the spacecraft angular velocity vector to the direction opposite to the orbit angular rate, so that the pitch axis becomes parallel to the orbit normal, for a positive asymptotic value of the pitch rate, or points in the opposite direction, for a negative pitch rate.

The article is organized as follows: a quick outline of system dynamics and of the Y-dot control law is provided in Section 2. After a review of a few relevant non-linear system stability theorems, a proof for the stability of the pure spin condition around the pitch axis is given in Section 3. The validity of the results, together with expected performance from the control approach are extensively tested and discussed by means of numerical simulation, discussed in Section 4. A section of conclusions ends the paper.