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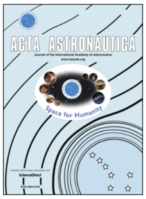

Bending analysis of embedded carbon nanotubes resting on an elastic foundation using strain gradient theory

Bekir Akgöz*, Ömer Civalek

Akdeniz University, Civil Engineering Department, Division of Mechanics, 07058 Antalya, Turkiye

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ABSTRACT

In this study, static bending response of single-walled carbon nanotubes (SWCNTs) embedded in an elastic medium is investigated on the basis of higher-order shear deformation microbeam models in conjunction with modified strain gradient theory. The governing differential equations and related boundary conditions are obtained by implementing a variational principle. The interactions between SWCNTs and surrounding elastic medium are simulated by Winkler elastic foundation model. The Navier-type solution is utilized to obtain an analytical solution for the bending problem of the simply supported embedded SWCNTs under uniform and sinusoidal loads. The influences of material length scale parameter-to-diameter ratio, slenderness ratio, loading type, shear correction factor and Winkler modulus on deflections of the embedded SWCNTs are discussed in detail. The present results illustrate that the bending behavior of SWCNTs is dependent on the small-size, stiffness of the elastic foundation and also effects of shear deformation, especially for smaller slenderness ratios.

1. Introduction

With the rapid advances in nanotechnology such as discovery of scanning tunneling microscope and atomic force microscope in 1980s, the miniaturized (small-sized) structures have a wide range of applications in micro- and nano-electro mechanical systems (MEMS and NEMS). Carbon nanotube is an allotrope of carbon and one of the key structures in the nanotechnology applications. Carbon nanotubes were firstly observed as multi-walled carbon nanotubes (MWCNTs) in a carbon arc discharge by Iijima, in 1991 [1]. Two years later, single-walled carbon nanotubes (SWCNTs) were discovered [2]. Carbon nanotubes have attracted many researchers, scientists and academicians, not only because of their unique electrical, thermal, optic, mechanical and chemical properties, but also because they are very promising materials in nanotechnology [3–6]. There are three individual geometries for carbon nanotube with related to how the graphene sheet is wrapped as armchair, zigzag and chiral (see Fig. 1).

It has been observed from some experiments that there is a size effect on the deformation behavior of the micro-/nano-sized structures [7–9]. However, experiments are very difficult and expensive in these scales because the high precision test devices are required. On the other hand, atomistic modeling such as molecular dynamic simulations is computationally expensive and requires a long time. Furthermore, it was a good idea that continuum mechanics models like elastic rod, beam, plate and shell can be used to investigate the mechanical characteristics of small-sized structures. However, in the absence of any intrinsic or material length scale parameters, classical continuum theories do not have the ability to predict the size-dependent deformation behavior of micro- and nano-

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* Corresponding author.
E-mail address: bekirakgoz@akdeniz.edu.tr (B. Akgöz).

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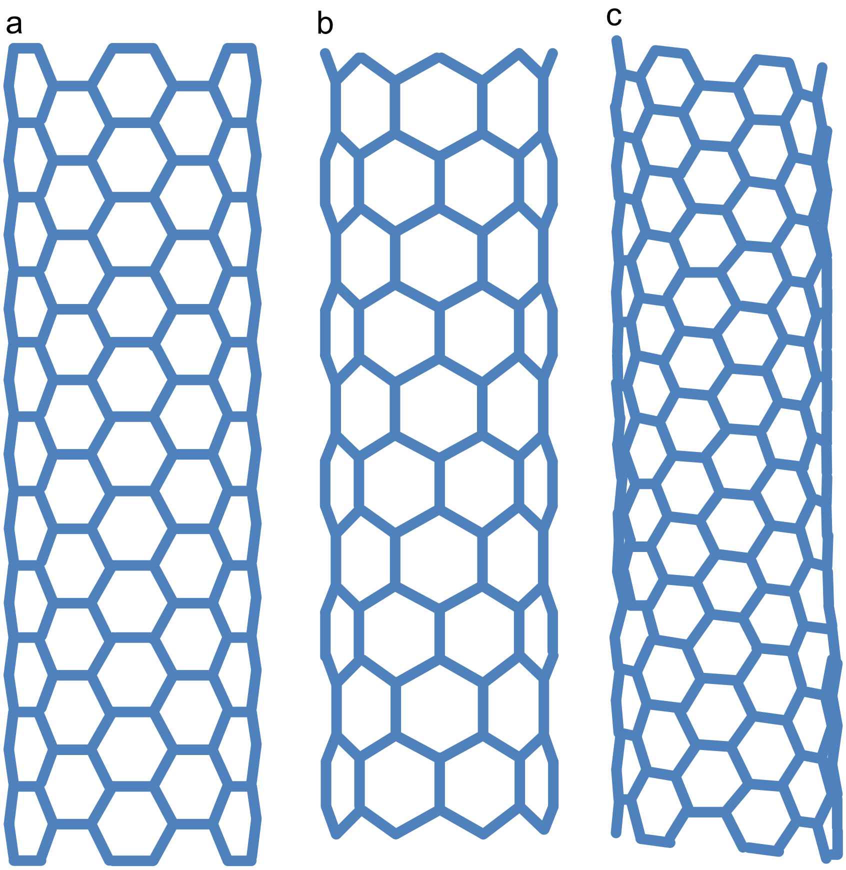

Fig. 1. Single-walled carbon nanotubes (a) Armchair, (b) Zigzag and (c) Chiral.

sized structures. In order to determine the mechanical responses of such structures, several non-classical continuum theories have been developed such as couple stress theory [10–12], micropolar theory [13], nonlocal elasticity theory [14,15] and strain gradient theories [16–19] Fig. 2.

Nonlocal elasticity theory is developed by Eringen [14,15] and according to this theory, the stress at any reference point in the body depends not only on the strains at this point but also on strains at all points of the body. This definition of the Eringen′s nonlocal elasticity is based on the atomic theory of lattice dynamics and some experimental observations on phonon dispersion. In this theory, the long range force about atoms is considered and thus internal scale effect is introduced in the constitutive equation. Static analysis of nano beams was performed by using the nonlocal Bernoulli–Euler model [20]. Wang and Liew [21] investigated the static bending responses of micro/nano structures based on nonlocal Bernoulli–Euler and Timoshenko beam models. Analytical solutions of bending, buckling and vibration for nanobeams on the basis of several nonlocal beam models were presented by Reddy [22]. A nonlocal shear deformation beam models for static and dynamic analyses of nanobeams were proposed by Thai [23,24]. Aydogdu [25] developed a general nonlocal beam theory for nanobeams.

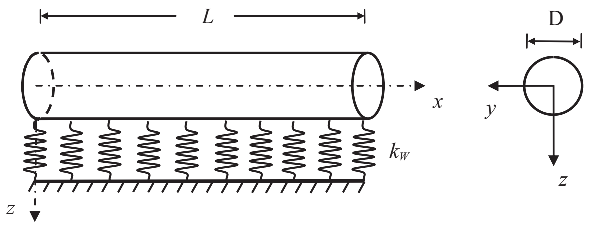

Fig. 2. Beam model of an embedded carbon nanotube.

Modified strain gradient theory (MSGT) was proposed by Lam et al. [8] in which strain energy density includes dilatation gradient vector, deviatoric stretch and symmetric rotation gradient tensors besides symmetric strain tensor. For linear elastic isotropic materials, the formulations and governing equations contain three additional material length scale parameters relevant to higher-order deformation gradients in addition to two classical ones. This popular theory has been extensively employed to develop size-dependent beam models. Kong et al. [26] and Wang et al. [27] developed Bernoulli–Euler and Timoshenko microbeam models for static bending and free vibration responses, respectively. Stability and bending analysis of Bernoulli–Euler microbeams with various boundary conditions was also carried out by the present authors [28,29]. Furthermore, Kahrobaiyan et al. [30],