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Approximating elliptic halo orbits based on the variation of constants

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ABSTRACT

A computational procedure is presented for approximating elliptic halo orbits about the collinear equilibrium points based on the variation of constants in the sense of the second-order approximation. Simulation results show that the approximate solutions agree well with the results of two-point boundary-value problems. The existence conditions of the elliptic halo orbits are also determined with respect to the mass ratio of the primaries and the eccentricity.

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1. Introduction

In the restricted three-body problem, the Lagrangian points consist of three collinear equilibrium points, called L_1, L_2, and L_3, and two triangular equilibrium points, called L_4 and L_5. The triangular equilibrium points are stable equilibria when the mass ratio between the primaries is small, whereas the collinear equilibrium points are unstable equilibria regardless of the mass ratio. Despite instability issues, orbits around the collinear equilibrium points are important for scientific missions, e.g., ISEE-3 [14], SOHO [5], and SPICA [11,19].

The orbital eccentricity of the Sun–Earth system is 0.0167. The circular restricted three-body problem (CR3BP), in which the orbital eccentricity is specified to be zero, is an applicable framework for analyzing halo orbits about the collinear Sun–Earth Lagrangian points. Farquhar considered the concepts of the Lissajous orbit and the halo orbit [6–8]. Richardson proposed a systematic method for obtaining analytical approximations of halo orbits by using the method of Lindstedt–Poincaré [12–14]. Breakwell and Brown [1] and Howell [10] numerically computed halo orbits.

The orbital eccentricity of the Earth–Moon system is 0.0549. The elliptic restricted three-body problem (ER3BP), in which the orbital eccentricity is specified to be above zero, is a framework that is more applicable to the analysis of halo orbits about the collinear Earth–Moon Lagrangian points. Brenton et al. computed halo orbits about triangular equilibrium points by using the canonical perturbation theory [2,3]. Hou et al. computed halo orbits by using the method of Lindstedt–Poincaré [9]. Campagnola et al. numerically computed halo orbits by using a continuation method from the computed orbits of the CR3BP with a period synchronous to that of the primaries [4]. A recent survey and several extensions on halo orbits can be found in [17,18,20,21]. An accurate first guess is essential for numerical computation because elliptic halo orbits about collinear equilibrium points are actually unstable orbits.

In this study, elliptic halo orbits about the collinear equilibrium points are analyzed based on the variation of constants. The equations of motion of the ER3BP are

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expanded in a Taylor series about the equilibrium points with respect to the eccentricity of the primaries. The zeroth terms, which correspond to linearized equations of the CR3BP, are selected to form an unperturbed system, and the remaining terms are regarded as the perturbation terms. Approximate solutions based on the variation of constants reflect the structure of solutions for the CR3BP. By expanding the perturbed system with respect to the eccentricity and the x-, y-, and z-components, the solutions can be sequentially approximated for given zeroth-order components of the approximate solutions. By selecting the initial conditions satisfying three conditions, such as the bounded, synchronization, and resonance conditions, second-order approximations of the elliptic halo orbits are obtained. In this way, the approximate solutions are systematically obtained. The effectiveness of the proposed computational procedure will be demonstrated by numerical simulations. Moreover, the existence conditions of the elliptic halo orbit are determined with respect to the mass ratio of the primaries and the eccentricity. These conditions will also be demonstrated by numerical simulations for the collinear points L_1 and L_2.

The paper is composed as follows: in Section 2, the perturbation equations are formulated for the ER3BP and analytical approximations of the elliptic halo orbits are obtained based on the variation of constants. A comparison between the approximations and numerical results is presented in Section 3, and the existence conditions of the elliptic halo orbits are also analyzed in Section 4. Finally, conclusions from this work are given in Section 5. An implementation of the method of the variation of constants is described in Appendix A and a comparison between the proposed method and the method of Lindstedt–Poincaré is presented in Appendix B.

2. Elliptic halo orbits

2.1. Second-order approximation of elliptic halo orbits

In the ER3BP, the first and the second bodies, called the primaries, move in elliptic orbits about their center of mass, and the third body, called the spacecraft, which has negligible mass, moves under the gravitational forces of the primaries. The mass ratio of the smaller primary to the total mass is given by $\rho = M_2/(M_1+M_2)$, where M_1 and M_2 denote the masses of the larger and the smaller primaries. The distance between the primaries is given by

$$r_{12}(\nu) = \frac{a(1-e^2)}{1+e\cos\nu}, \tag{1}$$

where a, e, and ν denote the semi-major axis, the eccentricity, and the true anomaly of the primaries, respectively. The unit distance is normalized so that the distance between the primaries is equal to one. Let $\boldsymbol{r} = [x\ y\ z]^T$ denote the normalized position vector of the third body in the rotating frame with the origin at the center of mass of the primaries, where the x-axis points from the larger primary to the smaller primary, the z-axis points in the direction of the angular velocity vector of the primaries, and the y-axis completes the right-handed coordinate system. The normalized equations of motion of the ER3BP are given by [16]

$$x'' - 2y' = -\frac{\partial U_e}{\partial x}, \tag{2}$$

$$y'' + 2x' = -\frac{\partial U_e}{\partial y}, \tag{3}$$

$$z'' = -\frac{\partial U_e}{\partial z}, \tag{4}$$

where the true anomaly of the primaries, ν, serves the independent variable, $()' = d()/d\nu$, $()'' = d^2()/d\nu^2$, and U_e is given by

$$U_e(\boldsymbol{r},\nu) = -\frac{1}{1+e\cos\nu}\left[\frac{1-\rho}{r_1}+\frac{\rho}{r_2}+\frac{1}{2}(x^2+y^2-e\cos\nu z^2)\right], \tag{5}$$

$$r_1 = \sqrt{(x+\rho)^2+y^2+z^2}, \tag{6}$$

$$r_2 = \sqrt{(x-1+\rho)^2+y^2+z^2}. \tag{7}$$

The Lagrange points consist of three collinear equilibrium points, called L_1, L_2, and L_3, and two triangular equilibrium points, called L_4 and L_5, of Eqs. (2)–(4) (see Fig. 1). Let x_{L_n} denote the x-component of the collinear equilibrium points about $L_n (n = 1, 2, 3)$. By substituting $y = z = 0$ into

$$\frac{\partial U_e}{\partial x} = 0, \tag{8}$$

x_{L_3}, x_{L_1}, and x_{L_2} are determined from the three solutions of

$$x\left(1-\frac{1-\rho}{|x+\rho|^3}-\frac{\rho}{|x-1+\rho|^3}\right)-\rho(1-\rho)\left(\frac{1}{|x+\rho|^3}-\frac{1}{|x-1+\rho|^3}\right)=0 \tag{9}$$

in increasing order. Eq. (9) coincides with that of the CR3BP and is independent of ν and e. This fact means that the ratio of the distance between the equilibrium points and that between the primaries is constant in the ER3BP. This fact means that the relative location of the collinear points and the primaries in the rotating frame are constant in the ER3BP.

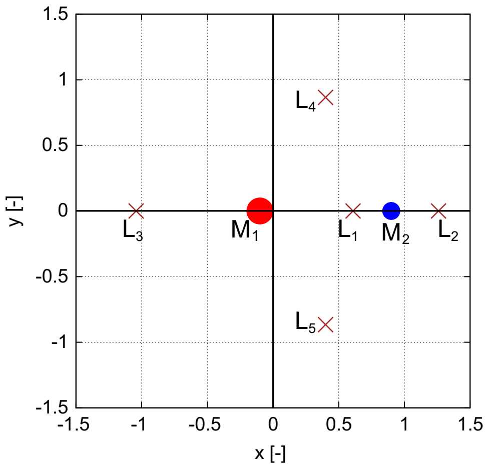

Fig. 1. The Lagrangian points, $L_1, \ldots, L_5$, in the rotating frame.

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