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Discrete event simulation methods applied to advanced importance measures of repairable components in multistate network flow systems

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ABSTRACT

Discrete event models are frequently used in simulation studies to model and analyze pure jump processes. A discrete event model can be viewed as a system consisting of a collection of stochastic processes, where the states of the individual processes change as results of various kinds of *events* occurring at random points of time. We always assume that each event only affects *one* of the processes. Between these events the states of the processes are considered to be constant. In the present paper we use discrete event simulation in order to analyze a multistate network flow system of repairable components. In order to study how the different components contribute to the system, it is necessary to describe the often complicated interaction between component processes and processes at the system level. While analytical considerations may throw some light on this, a simulation study often allows the analyst to explore more details. By producing stable curve estimates for the development of the various processes, one gets a much better insight in how such systems develop over time. These methods are particulary useful in the study of advanced importancez measures of repairable components. Such measures can be very complicated, and thus impossible to calculate analytically. By using discrete event simulations, however, this can be done in a very natural and intuitive way. In particular significant differences between the Barlow–Proschan measure and the Natvig measure in multistate network flow systems can be explored.

1. Introduction

Discrete event models are frequently used in simulation studies to model and analyze pure jump processes. For an extensive introduction to discrete event models we refer to [7]. A discrete event model can be viewed as a system consisting of a collection of stochastic processes (the *elementary processes* of the system), where the states of the individual processes change as results of various kinds of *events* occurring at random points of time. We always assume that each event only affects *one* of the elementary processes. Between these events the states of the processes are considered to be constant.

Let $S(t)$ denote the state of a pure jump process at time $t \geq 0$, let $T_1 < T_2 < \cdots$ denote the points of time of the events affecting the process, and let $T_0 = 0$. We assume that $S(t)$ can be written in the following form:

$$S(t) = S(0) + \sum_{k=1}^{\infty} I(T_k \leq t) J_k, \quad t \geq 0, \tag{1}$$

where $I(\cdot)$ denotes the indicator function, and J_k denotes the change in the state of the process at time T_k. This implies that the state function $S(t)$ is piecewise constant and right-continuous in t, with jumps at $T_1 < T_2 < \cdots$.

The fact that a pure jump process is right-continuous and piecewise constant in t is convenient during simulations. Hence, in order to keep track of how the process evolves and update the value of the state function, only the points of time where the events happen need to be considered.

The infinite sum in (1) indicates that the number of events occurring in the interval $[0, t]$ is unbounded. The possibility of having an infinite number of events in $[0, t]$, however, may cause various technical difficulties. In particular, this may cause simulations to break down since an infinite number of events need to be generated and handled. To avoid these difficulties, we always assume that the number of events occurring in any finite interval is finite with probability one. A pure jump process satisfying this

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assumption is said to be *regular*. For some basic results on regularity we refer to [10].

Stationary statistical properties of a system can easily be estimated by running a *single* discrete event simulation on the system over a sufficiently long time horizon, or by working directly on the stationary probability distributions of the elementary processes. Sometimes, however, one needs to estimate how the statistical properties of the system evolve over time. In such cases it is necessary to run many simulations to obtain stable results. Moreover, one must store much more information from each simulation. A possible approach to this is sampling the system state at fixed intervals of time, and then use the mean values of the states at these points as estimates of the corresponding properties. Alternatively, one may apply a more advanced sampling procedure where process data between the sampling points are utilized as well.

In the present paper we use discrete event simulation in order to analyze a multistate network flow system of repairable components. In order to study how the different components contribute to the system, it is necessary to describe the often complicated interaction between component processes and processes at the system level. While analytical considerations may throw some light on this, a simulation study often allows the analyst to explore more details.

When a system is evaluated from a reliability or availability perspective, it is often of interest to identify the most important components in the system. Since the pioneer work of [3] several approaches to this problem have been suggested. See e.g., [1,2,15]. The present paper is based on a recent paper in this area [17] and extends [10, 19] covering the binary case. See also [20]. The main focus here is to show how advanced importance measures of repairable components in multistate network flow systems can be computed using discrete event simulations. This approach allows us to explore in a new and intuitive way how the different importance measures incorporate information about the different components of the system. Examples of contributions to the area of component importance in multistate systems are presented in [22,21,26,25] and some references therein. The latter paper provides an interesting application to the railway industry. Furthermore, a part of the recent book [12] is devoted to this area.

For convenience we have included a list of symbols in Table 5.

2. Multistate systems

Most of the reliability theory literature focuses on binary systems, i.e., systems with only two states: *functioning* or *failed*. See e.g., [2]. In many real life applications, however, systems have more than two states. Among the first papers introducing a general theory of multistate systems are [5,8,16]. Rather recent books in this area are [14,13,18].

A typical example of a multistate system is a *network flow system* where the state of the system may be defined as the flow capacity of the system. Depending on the number of functioning links in the system, this capacity varies between full capacity and zero capacity, but with several intermediate states as well. Flow networks are the topic of the very recent interesting book [24]. For a thorough treatment of the more fundamental theory of network systems and stochastic networks we refer to [4].

Motivated by this example we consider a multistate system with component set $C=\{1,\dots,n\}$. The state of the ith component at time $t\geq 0$ is denoted by $X_i(t)$, $i=1,\dots,n$. We also introduce the component state vector $\boldsymbol{X}(t)=(X_1(t),\dots,X_n(t))$. The components of the system may be in several different states ranging from the *perfect functioning state* down to the *complete failure state*. We let $S_i=\{s_{i0},s_{i1},\dots,s_{ir_i}\}$ denote the set of possible states for the ith component, $i=1,\dots,n$. We assume that the states are ordered so that $s_{i0}<s_{i1}<\cdots<s_{ir_i}$. In particular s_{i0} denotes the complete failure state, while s_{ir_i} denotes the perfect functioning state. For simplicity we assume that we always have $s_{i0}=0$.

Here we assume that the components are *repairable*. The life cycle of the ith component starts out with the component being in its perfect functioning state, s_{ir_i}. Then the state of the component degrades through *all* intermediate states, and ends up in the complete failure state s_{i0}. After that the component is repaired back to its perfect functioning state again, and a new life cycle begins. In order to model this we introduce the following random variables:

$$U_{ij}^{(k)}=\text{The } k\text{th time spent by the } i\text{th component in state } s_{ij}, \tag{2}$$

where $i=1,\dots,n$, $j=0,1,\dots,r_i$ and $k=1,2,\dots$. All these random variables are assumed to be stochastically independent. This implies in particular that the component states $X_1(t),\dots,X_n(t)$ are independent for each $t\geq 0$. Furthermore, we assume that $U_{ij}^{(1)}$, $U_{ij}^{(2)},\dots$ are identically distributed with an absolutely continuous distribution with a positive mean value $\mu_{ij}<\infty$, $i=1,\dots,n$, $j=0,1,\dots,r_i$.

The system state at time $t\geq 0$ is denoted by $\phi(t)$ and we assume that this is uniquely determined by the component states, that is $\phi(t)=\phi(\boldsymbol{X}(t))$. The function ϕ will be referred to as the *structure function* of the system.

In the present paper we will focus our attention on a specific class of multistate systems. The main reason for this is that this enables us to establish an efficient representation of the system which will be needed in the calculations. More specifically, we assume that the system state is defined relative to a family of non-empty sets of components, $C_1,\dots,C_c$, where $C_r\subseteq C$, $r=1,\dots,c$. We assume that all the sets are unique, and that no set is a proper subset of another. We refer to $C_1,\dots,C_c$ as the *minimal cut sets* of the system. If a component i is a member of a minimal cut set C_r, its *contribution* to this minimal cut set at time t is given by its state at this time. The state of a minimal cut set C_r at time t, referred to as the *capacity* of C_r, and denoted by $\xi_r(t)$, is then given by

$$\xi_r(t)=\sum_{i\in C_r} X_i(t),\quad r=1,\dots,c. \tag{3}$$

The structure function, $\phi(t)$, is then assumed to have the following form:

$$\phi(\boldsymbol{X}(t))=\min_{1\leq r\leq c}\sum_{i\in C_r} X_i(t)=\min_{1\leq r\leq c}\xi_r(t). \tag{4}$$

Thus, we see that the state of the system can be interpreted as the state of the minimal cut set with the lowest capacity. This property is motivated by the well-known max-flow-min-cut theorem for calculating the flow capacity of a network. See [6]. In fact, by this theorem it follows that every network flow system can be represented by a model of the form (4). A multistate system where the structure function can be represented as in (4) is called a *flow type multistate system*.

As an example of a flow type multistate system we consider the network shown in Fig. 1. The components of the system are the five edges labeled $1,\dots,5$. The network represents a system transporting a flow of say oil from the source node A to the terminal node B. We assume that each edge has a maximum flow capacity of two units per second. Under certain circumstances, however, the capacity may drop to just one unit. This is reflected by letting the states of the components be defined as the corresponding flow capacities. That is, we interpret $X_i(t)$ as the number of units of oil that can flow through the ith edge at time t, and assume that $X_i(t)\in S_i=\{0,1,2\}$, $i=1,\dots,5$.

The system state is defined as the amount of oil that can be transported from A to B. In order to determine this we start out by