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An analytical method for out-of-plane relative equilibrium formation using inter-spacecraft forces

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ABSTRACT

For out-of-plane relative equilibrium of a two-spacecraft formation achieved by internal forces, this paper proposes a scheme which complies with the characteristics of angular momentum conservation to analyze such a formation over the entire parameter space. By purposely rewriting one of the equilibrium condition, this paper defines a modification of the set of equations for the out-of-plane equilibrium configurations. Thereby, the relative equilibrium can be solved analytically in the whole multidimensional real number field through these equations. Numerical examples are also included to provide some insight about the equilibrium conditions. Moreover, the results from derivation and illustrative examples disclose a novel picture of the out-of-plane relative equilibrium.

1. Introduction

Over the past decade, several types of formation flying using inter-spacecraft forces have been explored. The proposed inter-spacecraft forces range from contacting force to non-contacting force. Bae established a hybrid propulsion using photonic laser thrusters and tethers for precision formation flight [1]. King and Schaub et al. proposed the concept of Coulomb force and studied its application to space mission of satellite formation [2,3]. Miller and Scchweighart et al. considered a group of vehicles in orbit, which change or maintain their relative separation and orientation through electromagnetic interaction [4,5]. Other inter-spacecraft forces concepts are flux pinning interaction between a magnet field and a superconductor [6], and liquid droplet thrusters [7]. All of these approaches for formation flying apply the internal force between spacecraft to modify their separate natural orbit motion and form specific relative movement. Without loss of generality, this paper considers the case of two spacecraft formation with line-of-sight internal force between them.

Two-spacecraft formation in a central gravitational field can achieve two types of static relative configuration with the aid of inter-spacecraft forces [8,9]: great-circle relative equilibrium (GCRE) and nongreat-circle relative equilibrium (NGCRE). Relative equilibrium with the two spacecraft allocated along the regular direction (orbit radial, tangential and normal direction) is called great-circle equilibrium. Then, the attitude motion of the dumbbell representing the two-spacecraft formation is decoupled from the orbit motion, and the orbit of the formation center of mass is a great circle centered at the attraction center. According to the resulting GCRE and NGCRE solutions in Refs. [8,9] and the theorem 2 in Ref. [9], the masses of the spacecraft should be unequal for NGCRE, while such a condition is not required for GCRE. In NGCRE the orientation of the two spacecraft in a NGCRE is along an arbitrary direction. In this case, the orbit-attitude motion of the formation is highly

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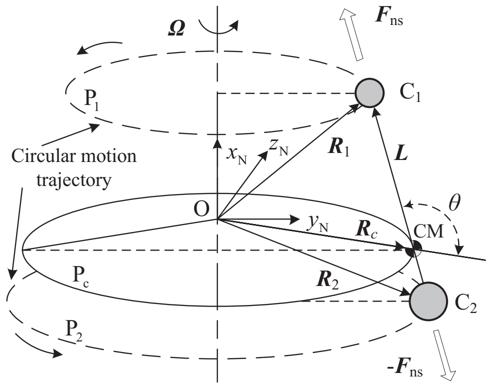

Fig. 1. Scenario of internal forces connected two spacecraft formation with unequal masses.

nonlinear and fully coupled, and the orbital radius vector of the formation center of mass traces a cone (see Fig. 1) rather than a great-circle. These complex properties therefore make the NGCRE problem solution quite challenging. Moreover, NGCRE is a common problem faced by many kinds of formation, such as Coulomb formation [3,10], electromagnetic formation [4,5], liquid droplet thrusters [7], and Flux-Pinned Spacecraft Formation [6].

Ref. [9] proves the existence of NGCRE for a massless spring-connected two masses system (dumbbell) by applying the principle of symmetric criticality. However, the computation of equilibrium is numerical, and is valid under the following restriction: the orbital radius of the formation center of mass (denoted asr_c) is much larger than the distance between spacecraft (denoted as l). Along with the development of the concepts of inter-satellite forces, most researchers have studied the relative equilibrium of proximity formation using the CW equation or linearized model under the assumption that the formation center of mass moves in a circular Keplerian orbit [3,6,11–14]. By employing a non-canonical Hamiltonian formulation of the relative motion, Schaub et al. [15] obtained the necessary equilibrium conditions for Coulomb formation with full nonlinear gravitational potential function being retained. Refs. [16,17] considered the spacecraft in a formation as linked by virtual joint, then the control force needed to precisely maintain an equilibrium configuration is turned into the constraint forces of each joint. Larsen et al. [18] investigated the modeling of satellites coupled tightly using tethers, and used a topology description to automate the formulation of the general equations for the motion of each satellite in a formation. Their paper also considered two spacecraft that are coupled with weaker inter-spacecraft forces, and the equilibrium solution therefore requires an accurate description of the relative velocities and accelerations of the spacecraft. More recently, Inampudi and Schaub [8] stated that the method in Ref. [9]is applicable for all elastic forces, and extended such method to a formation in a restricted three-body system. Overall, due to the nonlinear coupled motion and limitation of research viewpoint, the previous solution methods for NGCRE, relying on linearization or numerical calculation, are valid only in a partial value-range of parameter space.

The present paper aims at developing a new method for NGCRE that is free from the above drawbacks, and designs an analysis scheme as follows. First, we modify an equation for equilibrium conditions in Ref. [9] to make it an analytically solvable expression. Then, we create a rotating reference frame according to the formation's angular momentum conservation law, and set a list of generalized coordinates which have practical physical significance. Next, considering that NGCRE's motion is equal to two displaced orbits, the kinematic equation for the equilibrium formation system is expressed as dynamic balance equations. Finally, integrating all of these equations with a relationship of formation geometry, we obtain a set of new equations for nongreat-circle relative equilibrium conditions. We also demonstrate the capacity of explicit evaluation over entire valid parameter space by resolving the equilibrium solutions and configuration parameters. Furthermore, distribution properties of NGCRE are disclosed through calculation and in-depth derivation by taking advantage of these new equations.

This paper is organized as follows. Section 2 addresses the NGCRE problem and a previous approach for NGCRE. Section 3 presents the analysis strategy and formulates the equilibrium equations, and then validates them. By using these equations, we analytically solve and then analyze NGCRE in Section 4. Section 5 states the conclusions of this study.

2. NGCRE statement

2.1. Mathematical definition of NGCRE

A simple NGCRE formation containing two point masses with mutual internal forces between them is shown in Fig. 1. Mark O is the center of the inverse square field and also the origin of the inertial frame, denoted as N: $\{x_N, y_N, z_N\}$.m_i and $\boldsymbol{R}_i$ are the mass and the position vector of spacecraft i ($i=1, 2$), respectively. Therefore, $\boldsymbol{L}=\boldsymbol{R}_1-\boldsymbol{R}_2$ is the relative position from Spacecraft 2 (denoted by C_2) to Spacecraft 1 (denoted by C_1). Let $\boldsymbol{F}_{ns}$ be the line-of-sight inter-spacecraft force acting on Spacecraft 1, and then the inter-spacecraft force acting on Spacecraft 2 is $-\mathbf{F}_{ns}$.

In the absence of environmental perturbations, one constant internal force $\boldsymbol{F}_{ns}$ just cancels perfectly the differential gravitational forces acting across the formation. Then, the behavior of the spacecraft system is equivalent to a rigid body in orbit. In other words, the relative equilibrium is achieved by applying constant internal forces, although the stability of such relative equilibrium is not considered in the present paper. In the case of NGCRE, Spacecraft 1 and Spacecraft 2 are moving synchronously with the same orbital angular velocity $\boldsymbol{\Omega}$ on two parallel planes P_i ($i=1, 2$), which displace each other above and below the common center of gravity. As shown in Fig. 1, the formation system is viewed from the frame N.

For an observer traveling with a uniformly rotating orbital frame located at the origin O, Spacecraft 1 and Spacecraft 2 are stationary. Accordingly, the mathematical definition of nongreat-circle relative equilibrium is: for all $\boldsymbol{R}_i(t)$ and time, the equation $\boldsymbol{R}_i(t)=\exp(\Omega^{\times}t)\boldsymbol{R}_i(0)$ is invariant under the rotation action, and the formation center of mass (CM) satisfies $\boldsymbol{R}_c\cdot\Omega\neq 0$, where $\boldsymbol{R}_i(0)$ is an initial constant vector in the inertial frame N. The matrix exponential $\exp(\Omega^{\times}t)$ is an orthogonal rotation transformation