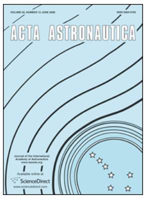
Acta Astronautica 89 (2013) 107–120

Contents lists available at SciVerse ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro



A dynamical approach to precision entry in multi-body regimes: Dispersion manifolds

B.G. Marchand [a,b,*], S.K. Scarritt [c], T.A. Pavlak [a], K.C. Howell [a]

[a] *Purdue University, 701 W. Stadium Ave., West Lafayette, IN 47907, United States*
[b] *Decisive Analytics Corporation, 1235 South Clark Str., Ste. 400, Arlington, VA 22202, United States*
[c] *The University of Texas at Austin, 210 E. 24th St, Austin, TX 78712, United States*

ARTICLE INFO

Article history:
Received 18 August 2011
Received in revised form
15 February 2013
Accepted 20 February 2013
Available online 13 March 2013

Keywords:
Multi-body
Precision entry
Manifold
Dynamical systems
Interplanetary
Targeting

ABSTRACT

The identification of trajectories that target a precise location and approach vector during planetary entry is sensitive to the quality of the startup arc supplied to iterative path planning and guidance algorithms. These sensitivities are especially evident when multi-body effects are significant; low-energy spacecraft trajectories that dwell near the gravitational boundary of two bodies, for instance, are more susceptible to third-body effects. Dynamical sensitivities are also significant when maneuvers are scheduled within a region of space susceptible to multi-body effects. The present study considers precision entry targeting from the perspective of the multi-body problem.

1. Introduction

Precision entry targeting, in this study, refers to the identification of all maneuvers necessary to guide the vehicle to a specific location on a rotating body with a pre-specified approach vector at entry interface. In the present study, precision entry targeting is considered in the context of the perturbed restricted three body problem. The primary bodies under consideration are the Earth and the Moon and the problem is defined as "restricted" because the mass of the spacecraft is assumed to be significantly smaller than that of the primaries. Furthermore, "perturbed" suggests that the motion of the primary bodies is consistent with an ephemeris model; that is, the position and velocity of the primaries are derived from planetary ephemerides. In the perturbed restricted three-body problem, targeting processes (optimal or suboptimal) are sensitive to the quality of the startup solution provided. Generally speaking, gradient based targeting algorithms, whether optimal or suboptimal, are not self-starting and thus depend on the availability of a reasonably accurate initial guess (i.e. startup solution). Since the dynamical model is not time invariant, the success of any targeting process can be sensitive to both the temporal and spatial scheduling of deterministic maneuvers along the path.

Startup arcs employed in iterative path planning and guidance algorithms often rely on conic or patched-conic approximations for the identification of startup solutions. Of course, two-body approximations are not always sufficiently accurate for trajectory design in multi-body regimes. This is particularly true when the path of the vehicle is expected to escape the Hill sphere with a relatively low energy level. As the vehicle transitions through a dynamically sensitive region, the gravitational influence of the primaries and the perturbing bodies can

---

[*] Corresponding author at: Decisive Analytics Corporation, 1235 South Clark Str., Ste. 400, Arlington, VA 22202, United States. Tel.: +1 512 627 4785; fax: +1 512 233 2349.
E-mail address: belinda.marchand@dac.us (B.G. Marchand).

0094-5765/$ - see front matter © 2013 IAA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.actaastro.2013.02.015

introduce significant deviations from the intended path. This, in turn, affects the efficiency of iterative path planning and guidance algorithms that seek to fine tune the transfer parameters to achieve a precise set of entry conditions at a specific body within a multi-body system. It also affects the subset of phase space explored by the algorithms in attempting to identify a feasible transfer.

The manifestation of these dynamical sensitivities is easily observed in the three-maneuver trans-Earth injection (TEI) sequence originally envisioned to transfer the Orion vehicle from low lunar polar orbit to a specified Earth arrival condition [1]. The initial design of this sequence, rooted in two-body analysis, is illustrated in Fig. 1(a) and the associated maneuver schedule is shown in Fig. 1(b). The first maneuver (TEI-1), in Fig. 1, seeks to raise the apoapsis of the initial lunar orbit. The second maneuver (TEI-2) executes a change in orbital inclination. The third and final maneuver (TEI-3) injects the spacecraft into its final return path.

The present investigation offers some preliminary insight into the precision entry problem in multi-body regimes. Initially, this is accomplished by generating an ensemble of dispersion trajectories associated with a representative set of possible entry interface states relative to the rotating target body. This ensemble of dispersion trajectories represent a subset of a “manifold” surface associated with a particular entry interface state. That is, this surface represents the subset of the dynamical flow that converges onto the vicinity of the specified entry state. The perturbed restricted three-body problem serves as the initial framework for this analysis. A fundamental understanding for the interaction between the dispersion manifolds and the Hill sphere is sought. The goal is to assess entry constraint coupling and sensitivities which may affect the process by which startup arcs, for targeting, guidance, and optimization processes, are subsequently identified.

2. Background

The Hill sphere is defined in the synodic rotating frame of the circular restricted three-body problem (CR3BP) [2]. In this frame, it is assumed that the primaries evolve along circular orbits about their common center of mass. The rotating x-axis is directed from the larger to the smaller primary such that both remain equidistant along that line for all time. The z-axis is normal to the plane of their orbits while the y-axis completes the right-handed triad. The Hill sphere itself is centered at the smaller of the two primary bodies. In the Earth–Moon system, the radius of the sphere (r_s) is approximately determined as [3]

$$r_s = a\sqrt[3]{\frac{\mu_{Moon}}{3\mu_{Earth}}}. \tag{1}$$

where μ_{Earth} and μ_{Moon} represent the gravitational parameters for the Earth and Moon, respectively, and a is the semi-major axis of the Moon's orbit around the Earth. In this system, the relative size of the Hill sphere, in relation to the Earth, Moon, and the libration points of the CR3BP, is illustrated in Fig. 2.

Since the Hill sphere is defined in the CR3BP, where the primary bodies remain equidistant for all time, identifying an equivalent Hill sphere in the general perturbed three-body problem requires that all trajectories be transformed into a properly scaled set of coordinates. For instance, if the Earth–Moon dynamics are derived from ephemeris information, the inertial (I) position and velocity vectors of the Moon, with respect to the Earth, in terms of inertial coordinates are given by $\boldsymbol{r}_I^{EM}$ and ${}^{I}\boldsymbol{v}_I^{EM}$, respectively. These vectors are then used to define an instantaneous synodic rotating frame (R) in terms of unit vectors $\hat{r}_1$, $\hat{r}_2$, and $\hat{r}_3$ where

$$\hat{r}_1 = \frac{\boldsymbol{r}_I^{EM}}{\|\boldsymbol{r}_I^{EM}\|}, \tag{2}$$

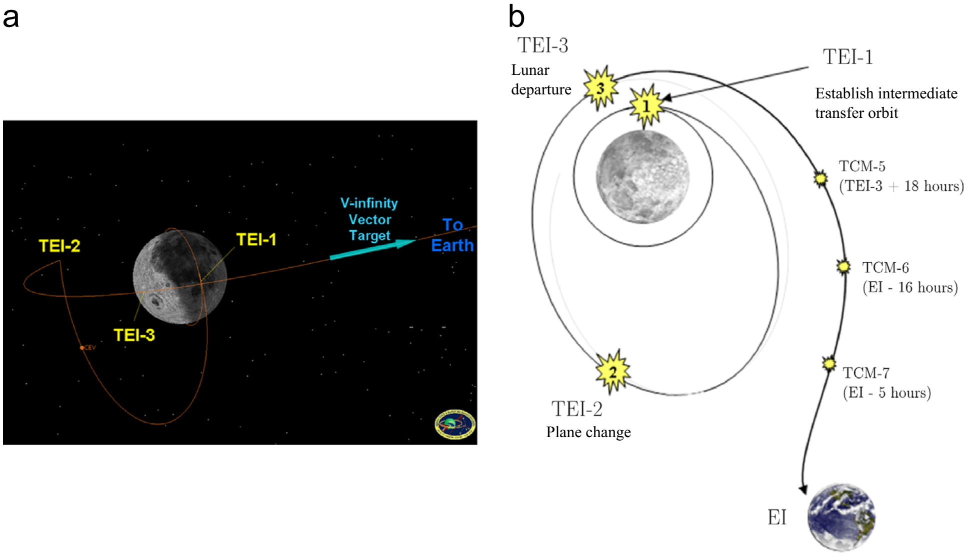


Fig. 1. Orion trans-Earth trajectory and maneuver schedule. (a) Return trajectory. (b) Maneuver schedule.