Aerospace Science and Technology ••• (••••) •••–•••

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Aerospace Science and Technology

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Three-dimensional cooperative guidance and control law for multiple reentry missiles with time-varying velocities

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ARTICLE INFO

Article history:
Received 10 December 2016
Received in revised form 8 May 2018
Accepted 8 July 2018
Available online xxxx

Keywords:
Multiple reentry missiles
Cooperative guidance
Partial integrated guidance and control
Dynamic surface control
Extended state observer

ABSTRACT

A three-dimensional partial integrated guidance and control law is proposed for cooperative flight of multiple hypersonic reentry missiles with uncontrollable and time-varying velocities. As for the problem of multiply reentry missiles with varying velocities attacking the target synchronously, a cooperative scheme is presented by adjusting the lateral pre-setting angle of velocity. In addition, a partial integrated guidance and control method with a two-loop controller structure is designed to realize the proposed cooperative scheme. Considering the saturation of the velocity slope angle and the unknown uncertainty, the two-loop three-channel controller of each reentry missile is designed based on dynamic inverse theory, dynamic surface control theory and extended state observer. The stability of the closed-loop system is demonstrated by Lyapunov theory. Simulation results verify the effectiveness and superiority of the proposed guidance and control law.

1. Introduction

For the last few decades, hypersonic reentry missile has attracted a lot of attentions due to its high velocity, far flight distance and good combat performance [1,2]. Many researches have been conducted on such problems of hypersonic reentry missile as aerodynamic characteristics, structure, materials and trajectory optimization [3–5]. With the rapid development of missile defense system, the demand of cooperative combat of multiple missiles is increasing. Many researches on formation flight and simultaneous attack of multiple subsonic missiles have been carried out [6–9]. However, the velocity of each missile is supposed to be constant. The assumption has no application to the multiple hypersonic reentry missile system since the velocity of each hypersonic reentry missile is changing in a wide range. These classical cooperative guidance laws are not available for the simultaneous attack of the multiple reentry missile system. Therefore, it is necessary to deal with the problem of the cooperatively simultaneous attack of multiple reentry missiles with time-varying velocities.

Many efforts on simultaneous attack for multiple missiles can be classified into two categories. The first category is individual homing, in which missiles realize simultaneous attack by selecting common flight time t^* in advance. In this category, there is no requirement to establish the communication topology among multiple missiles and many researchers mainly focus on designing the guidance law with impact-time constraint [9–13]. Many advanced control techniques, such as optimal control, first-order sliding mode control and nonsingular terminal sliding mode control, have been applied into the cooperative guidance law with impact-time constraint. However, one of the disadvantages of this category is that it is difficult to give a reasonable common flight time t^* in advance, especially for diifferent types of missile attacking moving target. Additionally, if we change the target, since there is no communication and missiles can not achieve cooperation autonomously, the guidance law will fail.

The other category overcomes the shortcoming of the first one by introducing real-time communication topologies in the multi-missile system, i.e., missiles communicate with each other to achieve simultaneous attack. By following this, a cooperative proportion navigation (PN) law was proposed in [14], where the navigation gain was changed based on the onboard time-to-go of own and the times-to-go of the other missiles. In [15], a suboptimal rendezvous time was decided based on the times-to-go of all missiles. Then the suboptimal rendezvous time was broadcasted to all of the missiles that would adjust their flight to realize the time. In these guidance laws, missiles use centralized communication topology, which means there is a centralized unit to collect the

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https://doi.org/10.1016/j.ast.2018.07.011
1270-9638/

Please cite this article in press as: X. Wang et al., Three-dimensional cooperative guidance and control law for multiple reentry missiles with time-varying velocities, Aerosp. Sci. Technol. (2018), https://doi.org/10.1016/j.ast.2018.07.011

Nomenclature

θ_t	Flight path angle of target
ψ_{vt}	Flight path deflection angle of target
$\theta_m(\theta)$	Flight path angle of missile
$\psi_{vm}(\psi_v)$	Flight path deflection angle of missile
γ_v	Velocity slope angle
α	Angle of attack
β	Angle of sideslip
δ_x	Deflection angle of aileron
δ_y	Deflection angle of rudder
δ_z	Deflection angles of elevator
X	Drag force
Y	Lift force
Z	Lateral force
P	Thrust
$c_y^{\alpha}, c_y^{\delta_z}, c_z^{\beta}, c_z^{\delta_y}$	Derivatives of aerodynamic coefficients
$m_x^{\beta}, m_x^{\delta_x}, m_x^{\omega_x}, m_y^{\beta}, m_y^{\delta_y}, m_y^{\omega_y}, m_z^{\alpha}, m_z^{\delta_z}, m_y^{\omega_z}$	Derivatives of aerodynamic moment coefficients
M_x, M_y, M_z	Rolling moment, yawing moment and pitching moment
$\omega_x, \omega_y, \omega_z$	Rolling rate, yawing rate and pitching rate
J_x, J_y, J_z	Moments of inertia of the axes of body coordinate system
m	Mass of missile
g	Gravity acceleration
S	Reference area
L	Reference length
q	Dynamic pressure

information of all missiles, form the command and then broadcast to all missiles. Besides, distributed cooperative guidance laws were investigated in [16–19] based on decentralized communication. Assuming that the missile can only collect the information from its nearest neighbor, a distributed coordination algorithm is proposed in [16] to enhance the engagement of the multi-missile network in consideration of obstacle avoidance. In [17], a two-stage guidance scheme for simultaneous attack from a group of missiles was proposed. In the first stage, a special distributed consensus protocol is designed to make all missiles asymptotically achieve the flight state consensus. A local sightline control law is applied to make missiles to independently reach the target in the second stage. In [18,19], the communication noisy and communication topology switching were considered during the attack process. All of these cooperative guidance laws are proposed in two-dimensional (2D) plane and the missile velocity is assumed to be constant. Furthermore, three-dimensional (3D) cooperative guidance laws were developed in [20] and [21], in which a distributed receding horizon control scheme and a two-stage salvo attack guidance law considering communication delay were presented, respectively. The guidance laws derived in [20,21] still can only be used to the missiles with constant velocity, i.e., missiles realize the cooperation only through adjusting their flight directions. In [22], without the assumption of constant velocity in the 3D cooperative guidance law, the cooperative velocity commands on the line-of-sight direction and on the normal direction of the line-of-sight direction are presented respectively. However, it is unavailable in many practical applications because the missile velocity was not easy to control. Besides, time-to-go used in most guidance law [9–15,18,20,22] is difficult to estimate precisely, especially when the missile velocity changes with flight time.

As for the problem of realizing the designed cooperative guidance law, it is common in many existing studies that the dynamic characteristic of each missile is supposed to be ideal. However, the dynamic feature of each missile directly determines the response time to the guidance command in practice, which may affect whether the designed cooperative guidance laws can satisfy the desired impact time constraint. To handle with this problem, integrated guidance and control (IGC) scheme is introduced in our study, which combines the guidance loop with control loop and considers the guidance law and the dynamic characteristic of missile simultaneously. IGC can eliminate the time lag inevitably existed in the conventional guidance and control design, and exploit fully the synergistic relationships between the separating subsystems. However, quick maneuvers in the IGC framework, which executes in a single-loop, may destabilize the rapid dynamics of the system due to the inherent time-scale separation between the faster and slower dynamics of missile [23]. To overcome such difficulty, the concept of partial IGC (PIGC) with a two-loop structure has been proposed and applied in many areas, such as impact angle control of missiles, formation flight and obstacle avoidance of unmanned aerial vehicles (UAVs) [24–28]. PIGC can preserve the inherent property of time-scale separation of missiles, and retain the benefits of the IGC method.

Inspired by the above analysis, this paper aims to the multiple hypersonic reentry missile system with uncontrollable and time-varying velocities to achieve cooperative salvo attack. Under the assumption of ideal centralized communication, a 3D cooperative guidance and control law is presented based on the use of lateral pre-setting angle of velocity and the concept of PIGC method. The main contributions of this paper are as follows: 1) a 3D cooperative guidance scheme is proposed based on the information of each missile's range-to-go, where the information of time-to-go is unnecessary. 2) considering many practical constraints and uncertainties, including velocity slope angle constraints, modeling errors, aerodynamic perturbations and external disturbances, a robust and stable two-loop three-channel PIGC controller is designed to realize the cooperative scheme; 3) the proposed cooperative PIGC law can be applied to the missiles with uncontrollable and time-varying velocities and achieve the flight-time cooperation and flight position cooperation simultaneously.

The remainder of this paper is organized as follows: Section 2 presents the model describing the relative motion between the missile and the target as well as the PIGC model of the missile. Section 3 describes the cooperative scheme, and designs the robust PIGC controller to realize the scheme. Section 4 exhibits a simulation performance analysis of the cooperative PIGC law and the conclusion is drawn in Section 5.

2. Problem formulation

In this section, the relative motion model between a hypersonic reentry missile and a target is established in 3D space. Besides, the dynamics of each attacker is also described.

2.1. Relative motion model of the missile and the target

Without loss of generality, the 3D engagement geometry between a missile and a target is shown in Fig. 1, where $OX_IY_IZ_I$ and $OX_sY_sZ_s$ represent the inertial coordinate system and the line-of-sight (LOS) coordinate system, respectively [29]. $\boldsymbol{r}_m$ and $\boldsymbol{r}_t$ are the position vector of the missile and the position vector of the target, separately. $\boldsymbol{r}$ denotes the vector of relative distance between the missile and the target, which is called as the vector of

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