Aerospace Science and Technology 80 (2018) 203–211

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Efficient numerical algorithm of profust reliability analysis: An application to wing box structure

Kaixuan Feng, Zhenzhou Lu [*], Chao Pang, Wanying Yun

Northwestern Polytechnical University, School of Aeronautics, Xi'an, Shaanxi 710072, China

A R T I C L E   I N F O

Article history:
Received 5 January 2018
Received in revised form 30 March 2018
Accepted 6 July 2018
Available online 10 July 2018

Keywords:
Profust reliability analysis
Structural safety
Profust failure probability
Subset simulation
Wing box structure

A B S T R A C T

In aerospace engineering, the reliability analysis technique attracts increasing attention from many structure designers. Compared with the conventional reliability analysis, the probability and fuzzy state assumption (profust) reliability analysis is proposed as a more comprehensive and objective theory to evaluate the structural safety. Because of great computational burden of the existing methods in estimating profust failure probability, an efficient numerical algorithm is developed in this paper. The proposed method is based on an equivalent transformation of the profust failure probability, then the profust failure probability can be rewritten as the form of a series of conventional failure probabilities which have similar constructions. The subset simulation method is employed to compute all the conventional failure probabilities by using a set of samples repeatedly. Next, this method is applied to estimate the profust failure probability of a wing box structure. The calculation results indicate that the proposed method can reduce the computational cost dramatically with acceptable precision.

1. Introduction

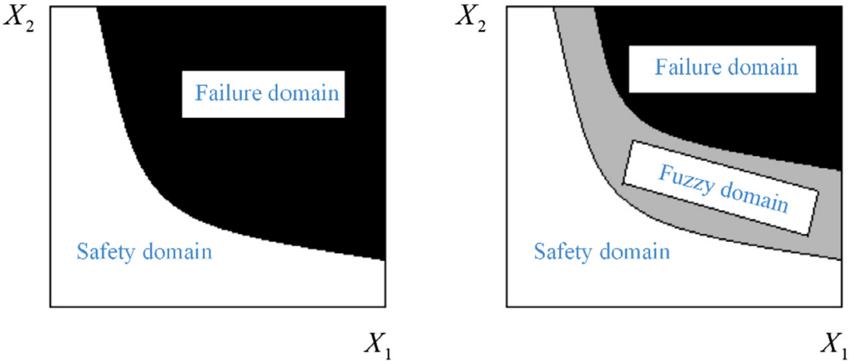
Along with the rapid development of computer technology, an increasing number of complex numerical models are proposed and developed for analyzing or predicting the performance of aircraft in various aspects of aerospace engineering, such as structural strength analysis [1,2], heat conduction prediction [3–8], aerodynamic analysis [9–13] and so on. In the conventional numerical simulation, the parameters of the calculation model are usually regarded as certain values. However, due to the manufacture errors, changing circumstances and incomplete knowledge, uncertainties of the input parameters widely exist which can cause the failure of structure or system to be random [1]. Thus, reliability analysis, which aims at measuring the safety degree of a system by considering the randomness of input parameters, has received a growing amount of attention in the past few decades, especially in the structure strength analysis field. Compared with the deterministic method, the reliability based structure design method is more accurate and, meanwhile, can provide an exact value of the failure degree, i.e. the failure probability, to the concrete structure.

The conventional reliability analysis technique, also be known as the probist reliability analysis method [14], is based on two basic assumptions. The first is the probability assumption for the model input, which indicates that the uncertainty of model input can be described by its probability density function (PDF) or cumulative distribution function (CDF). The second is the binary-state assumption for the output, which shows that there is a clear boundary between the safety domain and the failure domain. During the past few years, a great deal of computational methods for probist reliability analysis are proposed and developed, and they can be divided into three main categories, i.e., approximate analytical methods [15–17], numerical simulation methods [18–20] and surrogate model methods [21–23]. The approximate analytical method approximately estimates the probist reliability by computing some statistics moments of the model output, which is simple but significantly less accurate for highly non-linear models. As the most general and representative algorithm of the existing numerical simulation methods, Monte Carlo simulation (MCS) method is clear, exact and easy to implement. Nevertheless, in order to obtain a convergence solution, the crude MCS method needs a larger number of model evaluations, which is extremely time consuming especially for the finite element models. Thus, some improved numerical simulation methods are proposed by different scholars, such as the importance sampling method [19], line sampling method [20] and subset simulation method [24–26], which can reduce the calculation cost to some extent while the computational precision still keeps. The surrogate model aims at approximating the relationship between the random inputs and the model response with a small number of samples. By adopting an excellent

---

[*] Corresponding author.
E-mail addresses: 118392592026@163.com (K. Feng), zhenzhoulu@nwpu.edu.cn (Z. Lu), pang_c16@mail.nwpu.edu.cn (C. Pang), wanying_yun@163.com (W. Yun).

https://doi.org/10.1016/j.ast.2018.07.009



(a) Partition to the inputs domain in probist reliability analysis

(b) Partition to the inputs domain in profust reliability analysis

Fig. 1. Different partitions to the inputs domain in probist reliability analysis and profust reliability analysis.

strategic design of experiments, only a few numbers of model evaluations is needed in a surrogate model, thereof it can extremely improve the computational efficiency. But as pointed in Ref. [27], this method may be incapable of capturing the model behavior for the model which is primarily controlled by the high-order interactions.

In probist reliability analysis, structure designers usually define a threshold value, such as the limit displacement or the limit stress for the support structure, the extreme temperature for the thermal structure, so as to judge whether the structure is safe or not. If the model response is less than the threshold value, the structure is identified to be safe, otherwise, it is considered to be failure. As shown in Fig. 1(a), in probist reliability analysis, the safety domain and the failure domain are separated by a clear boundary, which is composed of the inputs whose responses are equal to the threshold value. However, for most practical aviation structures, there is no clear boundary between the failure and safety state, and the failure boundary is fuzzy in fact [28]. For instance, in aircraft structure design, the maximum displacement of a wing box structure should not be too large in case the structure is buckling or damaging. In probist reliability analysis, the limit displacement is usually defined and the relevant failure probability can be computed. However, with the gradual performance degenerating of the system, the cracks may expand gradually on the structure, so the structure may be damaged before its displacement coming up to the limit displacement. In this case, the binary-state assumption of the probist reliability framework is no longer applicable. Thereof, the profust reliability analysis theory is proposed at the right moment [14,29–31]. In the profust theory, the binary-state assumption is replaced by the fuzzy-state assumption, which admits the fact that the failure boundary is characteristic of fuzziness. As shown in Fig. 1(b), in profust reliability analysis, the inputs domain can be derived into three prats, i.e., the safety domain, the failure domain and the fuzzy domain. According to Fig. 1(b), it can be observed that the failure criterion is no longer clear, and some samples of model inputs (grey area) belong to neither failure domain nor safety domain, and this area can be called fuzzy domain. In fuzzy domain, the relevant outputs belong to the failure state or safety state at a certain membership degree. Hence, for the wing box structure, the profust reliability theory should be employed to measure its safety degree by considering both random input variables and fuzzy-state boundary condition.

Although the profust reliability analysis has been proposed for over three decades, and its computational methods are also developed by different scholars, some of these methods are lack of efficiency and some are difficult to achieve [32]. For instance, the conventional MCS method is computationally expensive especially for small failure probability problems. The fuzzy linear regression model proposed by Li et al. [33] has numerous adjustable coefficients which should be determined so as to solve the profust reliability problems. Thereof, in some cases, the solution of this model is not unique. In order to overcome the drawbacks of the existing computational methods, an efficient numerical algorithm based on subset simulation is proposed for estimating the profust failure probability in this paper. In this method, the profust failure probability is derived as a series of probist failure probabilities by using a mathematical equivalence transformation at first. Then, to further reduce the computational effort, a set of samples in the subset simulation method is repeatedly used in computing any element of the mentioned probist failure probabilities. In the proposed method, the total number of model evaluations is equal to the number of samples needed in a subset simulation.

This paper is framed in five sections. Section 2 establishes a finite element model of a wing box structure and then constructs its profust reliability analysis model. An efficient computational algorithm based on subset simulation is proposed in Section 3. In Section 4, the probist and profust failure probabilities of the wing box structure are estimated by adopting the MCS method and the proposed method. The results indicate that the proposed method can reduce the computational cost dramatically with acceptable precision. In addition, some specific suggestions are discussed to improve the profust reliability of the wing box structure. Conclusions are summarized in Section 5.

2. Profust reliability analysis model of the wing box structure

2.1. Finite element model of the wing box structure

The wing is the critical component to keep normal performance of the airplane as nearly all the lift is generated by the wing while the airplane is in flight. As the principle load-bearing structure of the wing, the wing box undertakes almost all the loads from whole wing. Hence, the structural safety of the wing box attracts increasing attention from many airplane designers [34,35]. The wing box consists of the ribs, spars, longitudinal strings and skin, and its structure is fairly complicated and diverse. In this article, the simplified model shown in Fig. 2 is constructed to replace the real wing box structure. This simplification does not affect the nature of the profust reliability problem, and it is easy to expand the proposed method to solve the profust reliability problems of various complex wing box structures or other large scale mechanical structures.

As shown in Fig. 2, the wing box structure is made up of 42 plates and 64 bars. According to their directions, 64 bars can be