Aerospace Science and Technology 79 (2018) 48–57

Contents lists available at ScienceDirect

Aerospace Science and Technology

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Efficient coning algorithm design from a bilateral structure

Chuanye Tang, Long Chen, Jianfeng Chen *

Automotive Engineering Research Institute, Jiangsu University, No. 301, Xuefu Road, Zhenjiang 212013, China

A R T I C L E I N F O

Article history:
Received 3 November 2017
Received in revised form 9 April 2018
Accepted 17 May 2018
Available online 21 May 2018

Keywords:
Efficient coning algorithm
Bilateral correction structure
Constraint relationship
Maneuver environment

A B S T R A C T

Many carriers in aerospace applications require high-precision strapdown inertial navigation system (SINS) for navigation. Under complex motion such as maneuver, vibration, etc., the performance of SINS algorithm needs to be paid special attention, since additional algorithm error can be induced due to complex motion. In order to improve the performance of SINS attitude algorithm, a bilateral coning algorithm is presented, which is based on a bilateral correction structure containing only one vector cross-product of which the undetermined coefficient is on both sides. In order to design the bilateral coning algorithm, the classical compressed algorithm coefficient is first given. Then the constraint relationship between the bilateral correction coefficient and the uncompressed correction coefficient is constructed. Further, it is shown that how to design the bilateral correction coefficient according to the constraint relationship. (The maneuver residual error based on the uncompressed correction structure is derived in Appendix A.) After the full analysis and simulation, the bilateral coning algorithm is verified to be very efficient in maneuver environment, for it has low algorithm throughput close to that of the compressed algorithm and high maneuver accuracy close to that of the uncompressed algorithm.

1. Introduction

At present, the inertial navigation technology as a fully autonomous navigation mode has been widely used in modern aviation [1], spaceflight [2], navigation and other fields, for carrier navigation [1–3], guidance [4] and control [5], etc. Generally, the precision of strapdown inertial navigation system (SINS) is mainly determined by device precision [6,7], initial system error [8–10] and algorithm accuracy. In complex environments such as maneuvering, vibrating and other environments, the error (caused by complex motion) of the algorithm used for the computation of carrier attitude, velocity or position will be prominent in high precision SINS.

The attitude calculation structure used for the modern SINS has not changed, since Jordan [11] and Bortz [12] introduced the two-stage structure [13] in which the calculation of equivalent rotation vector [14] is as the key point of attitude noncommutativity error compensation [13–24] (commonly referred to as coning correction now). The essence of coning correction is to approximate numerically the noncommutativity error from gyro data. To make coning correction as efficient as possible, two developing paths have been used for decades: one is to design a new coning correction structure instead of the existing coning correction structures, then to design the coefficients depending on the new structure; the other is to introduce a new approach to design the coefficients depending on an existing coning correction structure. The original coning correction structure is the uncompressed correction structure [15] developed from a two-sample correction structure earliest presented by Jordan [11]. All uncompressed coning algorithms as presented in [15–19] are based on the uncompressed correction structure. In order to improve the coning correction efficiency, Ignagni [20] proposed the compressed correction structure [15] which was simplified from the uncompressed correction structure based on the coning property that the cross-product of both angular increment samples is merely a function of the time interval between the two samples in the pure coning environment. All compressed coning algorithms as presented in [13,20,21] are based on the compressed correction structure. In order to take into account the algorithm performance and efficiency in maneuver environments, Tang et al. [22] proposed the half-compressed correction structure. In addition, Wang et al. [23] presented a high-order correction structure used for compensating the triple-cross-product term of attitude noncommutativity error. In all uncompressed coning algorithms and compressed coning algorithms, the coning algorithms proposed by Miller [16], Savage [13] and Song et al. [15] are representative.

The problem with traditional coning algorithms is that the traditional compressed algorithm [13,20] has low maneuvering accuracy, while the uncompressed algorithm [15] and the half-

* Corresponding author.
E-mail addresses: mhsmbyl@163.com, 406125415@qq.com (C. Tang), cjf@ujs.edu.cn (J. Chen).

https://doi.org/10.1016/j.ast.2018.05.027

compressed algorithm [22] have high throughput and low efficiency. In order to achieve both high accuracy and high efficiency of coning algorithm for complex environments, the authors of this paper concentrate on designing a new coning correction structure, and then designing a class of efficient coning algorithms for attitude computation. The new correction structure, denoted here as the bilateral correction structure where there is only one vector cross-product like that of the compressed correction structure, can be derived from the classical uncompressed correction structure based on some constraint condition. In order to design a class of coning algorithms based on the bilateral correction structure, the coefficients depending on the uncompressed correction structure need to be first designed under the constraint condition mentioned above. Then the coefficients depending on the bilateral correction structure can be calculated from the uncompressed coefficients of the last design based on the given constraint condition. It finally gives the new class of coning algorithms based on the bilateral correction structure. The new coning algorithm has been fully analyzed and simulated.

2. Coning algorithms basic

2.1. Coning correction structure

The rotation vector differential equation was introduced into inertial navigation by Bortz [14] as follows

$$\dot{\boldsymbol{\phi}} = \boldsymbol{\omega} + \delta\dot{\boldsymbol{\phi}},$$
$$\delta\dot{\boldsymbol{\phi}} = \frac{1}{2}\boldsymbol{\phi}\times\boldsymbol{\omega} + \frac{1}{\phi^2}\left(1 - \frac{\phi\sin\phi}{2(1-\cos\phi)}\right)\boldsymbol{\phi}\times(\boldsymbol{\phi}\times\boldsymbol{\omega}) \quad (1)$$

where $\boldsymbol{\phi}$ is an equivalent rotation vector representing an orientation or rotation of body, $\dot{\boldsymbol{\phi}}$ is the derivative of $\boldsymbol{\phi}$ to time, ϕ is the module of $\boldsymbol{\phi}$, $\boldsymbol{\omega}$ is an angular rate vector of body, and $\delta\dot{\boldsymbol{\phi}}$ is a non-commutativity rate vector.

For the SINS attitude updating, the equivalent rotation vector is generally calculated using a simple approximate form to the integral of the rotation vector differential equation. A common single-speed form [11–14] is as follows:

$$\boldsymbol{\phi}_l = \boldsymbol{\alpha}_l + \delta\boldsymbol{\phi}_l,$$
$$\boldsymbol{\alpha}_l = \int_{t_{l-1}}^{t_l} \boldsymbol{\omega}\, d\tau,$$
$$\delta\boldsymbol{\phi}_l = \frac{1}{2}\int_{t_{l-1}}^{t_l} \boldsymbol{\alpha}\times\boldsymbol{\omega}\, d\tau,$$
$$\boldsymbol{\alpha} = \int_{t_{l-1}}^{t} \boldsymbol{\omega}\, d\tau \quad (2)$$

where l denotes the attitude updating cycle, t is a time, t_{l-1} and t_l are respectively the beginning time and the ending time of the lth cycle, $\boldsymbol{\phi}_l$ is the rotation vector equivalent to the attitude change over the lth cycle, $\boldsymbol{\alpha}$ is the integral of $\boldsymbol{\omega}$ from time t_{l-1} to time t, $\boldsymbol{\alpha}_l$ is the integral of $\boldsymbol{\omega}$ from time t_{l-1} to time t_l, and $\delta\boldsymbol{\phi}_l$ is an approximate integral of $\delta\dot{\boldsymbol{\phi}}$ over the lth cycle, which denotes here the analytical form of coning correction.

In the implementation process of SINS attitude updating, $\boldsymbol{\phi}_l$ is generally approximated using a numerical method. Furthermore, $\boldsymbol{\alpha}_l$ is generally achieved by accumulating gyro output, and $\delta\boldsymbol{\phi}_l$ is generally approximated from gyro output using a numerical form. The forms [13,15,20,22] for calculating $\boldsymbol{\phi}_l$ with three existing coning correction structures for calculating $\delta\boldsymbol{\phi}_l$ are as follows:

Table 1
FTSc algorithm coefficients.

L	N	C_1	C_2	C_3	C_4
1	4	113/840	−26/840	3/840	
2	4	323/420	−26/420	3/420	
3	4	393/280	114/280	3/280	
4	4	214/105	92/105	54/105	
1	5	367/2520	−106/2520	21/2520	−2/2520
2	5	997/1260	−106/1260	21/1260	−2/1260
3	5	1207/840	314/840	21/840	−2/840
4	5	656/315	262/315	168/315	−1/315
5	5	1375/504	650/504	525/504	250/504

$$\boldsymbol{\phi}_l = \hat{\boldsymbol{\alpha}}_l + \delta\hat{\boldsymbol{\phi}}_l \quad (3)$$

$$\hat{\boldsymbol{\alpha}}_l = \sum_{k=N-L+1}^{N} \Delta\boldsymbol{\alpha}_k \quad (4)$$

$$\delta\hat{\boldsymbol{\phi}}_l = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \varsigma_{ij}\Delta\boldsymbol{\alpha}_i\times\Delta\boldsymbol{\alpha}_j \quad (5)$$

$$\delta\hat{\boldsymbol{\phi}}_l = \sum_{s=1}^{N-1} C_s\Delta\boldsymbol{\alpha}_{N-s}\times\Delta\boldsymbol{\alpha}_N, \quad C_s = \sum_{i=s+1}^{N}\varsigma_{i-s,i} \quad (6)$$

$$\delta\hat{\boldsymbol{\phi}}_l = \sum_{s=1}^{N-1} I_s\boldsymbol{\theta}_s\times\Delta\boldsymbol{\alpha}_{s+1}, \quad \boldsymbol{\theta}_s = \sum_{k=1}^{s}\Delta\boldsymbol{\alpha}_k \quad (7)$$

where Eq. (5), Eq. (6), and Eq. (7) are respectively called the uncompressed coning correction structure, the compressed coning correction structure, and the half-compressed coning correction structure, $\hat{\boldsymbol{\alpha}}_l$ is a numerical implementation of $\boldsymbol{\alpha}_l$ from gyro output, $\delta\hat{\boldsymbol{\phi}}_l$ denotes the numerical form of coning correction which is a numerical approximation to $\delta\boldsymbol{\phi}_l$, each $\Delta\boldsymbol{\alpha}$ is an angular increment (the integral of gyro sensed angular rate) sample over a fixed time interval T_k, the $\Delta\boldsymbol{\alpha}$s are adjacent and spaced sequentially forward in time, $\Delta\boldsymbol{\alpha}_{N-L+1}$ begins at time t_{l-1}, $\Delta\boldsymbol{\alpha}_N$ ends at time t_l, $\boldsymbol{\theta}_s$ is the angular increment over the time interval $[t_l - NT_k, t_l - (N-s)T_k]$ where T_k is the sample time interval, ςs, Cs, and Is are respectively the coefficients depending on the uncompressed structure of Eq. (5), the compressed structure of Eq. (6), and the half-compressed structure of Eq. (7), L is the number of angular increment sample selected to compute $\hat{\boldsymbol{\alpha}}_l$ in cycle l, N is the number of angular increment sample selected to compute $\delta\hat{\boldsymbol{\phi}}_l$ in cycle l.

2.2. Coning algorithm coefficients

For the design and evaluation of algorithms, the coefficients of several existing coning algorithms are directly given below. Any set of coefficients combined with the corresponding algorithm structure indicates some coning algorithm.

In Table 1 are the coefficients of the compressed algorithm designed using the frequency Taylor series method [20,21] (denoted as the FTSc method). In Table 2 are the coefficients of the compressed algorithm designed using the least minimum square [13] (denoted as the LMSc method). (Here $\dot{\Phi}(\Omega)$ is set to 1 when Ω is from 0 Hz to 100 Hz, and $\dot{\Phi}(\Omega)$ is set to 0 when Ω is greater than 100 Hz.) The compressed algorithms based on the compressed structure of Eq. (6) with the coefficients in Tables 1 and 2 are one of the basic coning algorithms, and will be used for the design of the new coning algorithm.

In Table 3 are the coefficients of the uncompressed algorithm designed using the Song et al. [15] method (denoted as the FTSuc method) from a part of the FTSc coefficients in Table 1. In Table 4 are the coefficients of the uncompressed algorithm designed using the Song et al. [15] method (denoted as the LMSuc method)