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Optimization of bounded low-thrust rendezvous with terminal constraints by interval analysis

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A B S T R A C T

A new indirect resolution method of an optimal control problem is proposed in this paper. And the optimization of the spacecraft low-thrust rendezvous with the fuel-minimum index to a safe region under the collision avoidance constraints is investigated. The objective is to minimize the fuel consumption in a power-limited low-thrust system, which leads to a bounded continuous control. The number of thrust arcs is unknown and the terminal positions in the rendezvous' safe region are unfixed for this optimization problem. The indirect resolution method of the optimal control employs deterministic interval analysis and gradient-based method to obtain the initial guess of the co-state variables. The interval analysis is used to sufficiently split, contract and clip the initial search space. And the gradient-based method is to determine the initial guess for each remained sub-space. Aiming at a low-thrust control system with the upper bound of acceleration of $5e^{-4}$ m/s^2, numerical results are given to validate the proposed optimization method.

1. Introduction

Electric propulsion system (EPS) for space missions has been well recognized [1] and successfully demonstrated in the mission of Deep Space 1 [2]. Due to high specific impulse, it produces low thrust, greatly decreasing the initial spacecraft mass. Therefore the Earth orbital transfer [3], rendezvous [4] and even future interplanetary missions using EPS can be accomplished efficiently. As a key technology, the optimizations of the low-thrust trajectory with large number of control arcs have been researched [5–10]. The control arc can be in the form of general continuous curve with the bound lower than the system's upper bound, i.e. the unbounded continuous low-thrust control [5,7]. Or it is the continuous approximate-square-wave with its bound equal to the system's upper bound, i.e. the bounded continuous low-thrust control [6]. The main resolution methods of the low-thrust trajectory optimization involve direct and indirect methods [11,12]. Direct methods solve an optimization problem via parameter discretization, parameter collocation and sequential quadratic programming [13]. The main disadvantage of direct methods for the low-thrust trajectory optimization is that the number of discretized parameter can be sufficiently large, which is improper for the low-thrust trajectory optimization. Indirect methods obtain the optimal solution of a problem via Hamiltonian boundary value problem (HBVP) and Pontryagin's maximum principle (PMP) [14]. The structures of all the control arcs satisfy the first-order optimality condition without any extra assumptions. And thus it is suitable for the low-thrust trajectory optimization. Nevertheless, a major drawback of indirect methods is the heavy reliance on a good initial guess and difficulty in optimizing the problem with a small convergence radius and sensitive initial co-state variables.

The basic methods to obtain the initial guess of the co-state variables include random guess and particle swarm optimization (PSO) [7,15]. Aiming at the orbital transfers subjected to the Earth and Sun's gravities [16,17], the control functions are unbounded continuous low-thrust control. The unbounded continuous low-thrust control involving a large convergence radius has been discussed before. And thus the initial co-state variables of the optimal low-thrust trajectories with energy-minimum index are easily yielded via these methods. However, it is difficult to yield the initial co-state variables of the optimal low-thrust trajectory control with fuel-minimum index via these methods. Because the control arc is square-ware with its bound equal to the upper bound of the low-thrust control system and the convergence radius is very small and the initial co-state variables are greatly sensitive. To solve this problem, a numerical continuation method named

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homotopic approach is combined to cope with the initial guess of the low-thrust trajectory optimization [18]. Particularly, the initial co-state variables of the optimal low-thrust trajectory control with energy-minimum index are firstly determined via the basic methods. And then the initial co-state variables corresponding to the fuel-minimum index are obtained via the homotopic approach based on the solutions of this problem with energy-minimum index. Obviously, these low-thrust, fuel-minimum problems solved via basic methods or homotopic approach are completely relied on an easily solved energy-minimum problem (optimal control solutions less than allowed maximum). Therefore, homotopic approach is effective to deal with the low-thrust, fuel-minimum trajectory optimization under the sufficient condition that the energy-minimum problem is an unbounded control problem. Note: "Unbounded" means "not reach its boundary" in this paper.

As for the low-thrust rendezvous, the fuel-minimum index is a half of the integral of the square of the control parameter in a power-limited system during the whole control period [19]. The control is unbounded or bounded continuous like the optimal control problem with energy-minimum index. Pardis derived the bounded low thrust trajectory with power-limited system [20]. Carter extended the work to the control system with upper and lower thrust bounds [21]. Guelman developed the power-limited unbounded or bounded thrust trajectories with the final constraint along the target-docking axis [6]. To satisfy the final constraint and obtain the optimal low-thrust trajectory, the fuel-minimum index is transformed into a fuel-state-optimal hybrid one. Consequently, the optimal low-thrust trajectories corresponding to the two indexes can be different under rendezvous constraint. Actually, a chase spacecraft (CS) within a certain constrained safe region (close to or station on the target-docking axis) can accomplish the final docking to a target spacecraft (several tens of meters away from the CS), using several target feature points [22]. Besides, due to the non-ignorable size of the target spacecraft (TS), the collision issue must be avoided in the rendezvous. In the safe region under the collision avoidance constraints, the bounded, low-thrust optimal rendezvous trajectory in power-limited system cannot be determined easily via aforementioned methods, because the control function is continuous approximate-square-wave with the bound equal to the upper bound of the low-thrust control system. And the convergence radius is quite small and the initial co-state variables are sensitive. Additionally, the homotopic approach is inappropriate for this optimization problem, because the optimal control solutions of the easily solved energy-minimum problem can be regarded as a bounded continuous control when the fuel-minimum problem is a bounded continuous control. Therefore, it is impossible to obtain the initial guess of the fuel-minimum problem via any numerical continuation methods.

In this paper, to overcome these drawbacks, a deterministic method obtaining the initial values of co-state variables in the low-thrust optimization problem is developed based on the interval analysis (IA) [23] and gradient-based method. Although it has been verified successfully only for the impulsive optimization of Lambert problem [24,25], it is essentially a deterministic optimization method and can solve the optimization problem in any nonlinear dynamical systems theoretically. And it is more effective for optimization problems with many constraints [24]. Therefore, a new estimation of the initial values of co-state variables of the low-thrust optimization problem will be presented in detail in this paper. Interval analysis used to solve a dynamic optimization problem (along the time 0–t) will be a new attempt. This method enriches the investigations of indirect resolution methods in the optimal control theory. And it may be helpful for the low-thrust space missions from the engineering viewpoint.

Particularly, this paper is organized as follows. Firstly, a new deterministic optimization method is introduced based on the IA and gradient-based method. Then, the fuel-minimum low-thrust rendezvous trajectory optimization with unfixed final position (reaching safe region) and collision avoidance is presented in the power-limited system. Subsequently, the low-thrust trajectory optimization in spacecraft rendezvous using the deterministic optimization method is discussed. Finally, the numerical simulations are implemented to validate the low-thrust trajectory optimization in spacecraft rendezvous with the fuel-minimum index.

2. Deterministic optimization using IA and gradient-based method

Combining with the branch and bound theory, the interval algorithm is developed into a deterministic optimization method [26]. However, it is really used to cope with the discontinuous and non-convex optimization problems and obtain a global minimum solution by Chen and Ma to impulsive Lambert problems [24,25]. In this paper, an optimization algorithm based on IA and gradient-based method is developed to deal with the low-thrust optimal control problem that is an integral multi-variable optimization problem.

2.1. IA

The optimization algorithm based on IA is a powerful tool to guarantee a global minimum solution to a nonlinear cost function. Firstly, given a large interval of each parameter of the nonlinear cost function, its interval outputs can be obtained by interval operations, which establish a boundary around the optimal solution of the nonlinear cost function [27]. The interval value and interval arithmetic are described as follows:

The interval value is defined as a sequential pair of real numbers $[x]$, which implies

$$[x] = [\underline{x}, \bar{x}] = \left\{ x | \underline{x} \le x \le \bar{x} \right\} \tag{1}$$

where $\underline{x}$ is the lower boundary of the interval value $[x]$, and $\bar{x}$ is the upper one. Obviously, an interval value can be an interval matrix (or an interval vector) shown as

$$[X] = \begin{pmatrix} [x_{11}] & [x_{12}] \\ [x_{12}] & [x_{22}] \end{pmatrix} \tag{2}$$

where $[x_{11}]$, $[x_{12}]$, $[x_{21}]$ and $[x_{22}]$ are the interval values.

The interval arithmetic can be regarded as a generalization or an extension of the real arithmetic. Similar to real arithmetic, the interval arithmetic involves basic operations as addition, subtraction, multiplication and division

$$\begin{cases} [x] + [y] = [\underline{x}, \bar{x}] + [\underline{y}, \bar{y}] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}] \\ [x] - [y] = [\underline{x}, \bar{x}] - [\underline{y}, \bar{y}] = [\underline{x} - \bar{y}, \bar{x} - \underline{y}] \\ [x] \cdot [y] = [\underline{x}, \bar{x}] \cdot [\underline{y}, \bar{y}] \\ \quad = [\min(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}), \max(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})] \\ [x]/[y] = [\underline{x}, \bar{x}]/[\underline{y}, \bar{y}] = [\underline{x}, \bar{x}] \cdot [1/\bar{y}, 1/\underline{y}] \quad 0 \notin [y] \end{cases} \tag{3}$$

The interval arithmetic also contains other operations, like trigonometric function, index function, function integration and differentiation, interval intersection etc. [19]. According to basic interval arithmetic, an interval function with interval variables $[x_1], \ldots, [x_n]$ can be expressed as

$$f\left([x_1], \ldots, [x_n]\right) = \Xi\left([x_1], \ldots, [x_n]\right) \tag{4}$$

where Ξ indicates the interval arithmetic of the interval function f with interval variables $[x_1], \ldots, [x_n]$.

Although a nonlinear cost function can be expressed and dealt with by an interval function, the overestimation of the interval

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