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A new efficient simulation method based on Bayes' theorem and importance sampling Markov chain simulation to estimate the failure-probability-based global sensitivity measure

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The failure-probability-based global sensitivity measure can detect the effect of input variables on the structural failure probability, which can provide useful information in reliability-based design. In this paper, a new efficient simulation method is proposed to estimate the failure-probability-based global sensitivity measure. The proposed method is based on the Bayes' theorem and importance sampling Markov chain simulation. The Bayes' theorem is used to provide a single-loop simulation method and the importance sampling Markov chain simulation is used to further reduce the computational cost. Compared to the traditional double-loop Monte Carlo simulation method, the proposed method requires only a single set of samples to estimate the failure-probability-based global sensitivity measure and its computational cost does not depend on the dimensionality of input variables. Finally, one numerical example and two engineering examples are presented to illustrate the accuracy and efficiency of the proposed method.

1. Introduction

In practical computational models for engineering structures, such as aeronautical engineering and civil engineering, uncertainty often arises from incomplete information [1–4], which leads to uncertain performances. To quantify the uncertainty of the model output and assess the risk, uncertainty analysis has been successfully and widely used in engineering [5–8]. An important content of uncertainty analyses is the reliability analysis [9–13], whose primary objective is the estimation of the failure probability. The failure probability can represent how likely the failure occurs, which can help us understand the safety level of a structural system. In reliability-based design, it is desired to get the influence of system parameters on the failure probability. Then the most influential parameters can be obtained so that it can provide useful information in risk-based decision making problems [14]. The reliability sensitivity analysis can help us measure the influence of system parameters on the failure probability [15]. In traditional reliability sensitivity analysis, the sensitivity of the failure probability is often measured by estimating the partial derivative of the failure probability with respect to the distribution parameters of random input variables [14,16–20]. This can be considered as local sensitivity analysis since it can only measure the effect of some statistical characteristics of input variables (such as mean and standard deviation) on the failure probability at nominal values. Therefore, it cannot detect the global effect of input variables on the failure probability in their whole uncertainty ranges, and cannot provide a global importance ranking of input variables [21].

To measure the global effect of random input variables on the failure probability in their entire distribution ranges, the global sensitivity analysis (GSA) is required [22]. The existent GSA methods, such as screening method [23–25], variance-based method [26–30] and moment-independent method [31–34], mainly focus on the models with real-valued continuous output. However, in reliability analysis, we are more interested in whether the system fails or not, which can be represented by the sign of model output. Then, the model output can be generally considered as a binary variable. Therefore, the GSA methods mentioned above cannot be used in reliability analysis directly [35]. To measure the global effect of random input variables on the failure probability and provide a global importance ranking of input variables, Cui et al. [36] proposed a failure-probability-based global sensitivity measure. This sensitivity measure is analogous to the moment-independent sensitivity measure proposed by Borgonovo [32], but it mainly focuses on the failure probability which is often related to the tail behavior of the distribution of model output. Compared to the traditional reliability sensitivity analysis which estimates

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the partial derivative of the failure probability with respect to the distribution parameters of input variables, the failure-probability-based global sensitivity analysis can measure the average effect of input variables on the failure probability in their entire distribution ranges.

The failure-probability-based global sensitivity measure is defined as the average difference between the unconditional failure probability and the conditional failure probability when certain input variables are fixed. It can reflect the average changes of the failure probability when the input variables are fixed. To estimate this sensitivity measure, the traditional Monte Carlo simulation (MCS) method requires a double-loop sampling procedure, in which the conditional failure probability requires to be calculated many times and the computational cost is dependent on the dimensionality of input variables. Therefore, this method is not efficient enough, especially for computational expensive and high dimensional problems.

In this work, we propose an efficient simulation method to calculate the failure-probability-based global sensitivity measure through Bayes' theorem [37,38] and importance sampling Markov chain simulation [39]. Based on Bayes' theorem, the original definition of the failure-probability-based global sensitivity measure can be represented as the area difference between the unconditional probability density function (PDF) and the failure-conditional PDF of input variable. Based on this representation, a new simulation method can be obtained, which requires only a single set of input samples. Then the importance sampling Markov chain simulation is utilized to further improve the computational efficiency. Compared to the traditional double-loop MCS method, the proposed method is more efficient, even the computational cost does not depend on the dimensionality of input variables.

The rest of this work is organized as follows. Section 2 gives a brief review of the failure-probability-based global sensitivity measure. Section 3 presents the new computational method of the failure-probability-based global sensitivity measure. In section 4, several examples are presented to illustrate the accuracy and efficiency of the new method. Section 5 gives the conclusions.

2. Review of the failure-probability-based global sensitivity measure

2.1. Definition of the failure-probability-based global sensitivity measure

Suppose the performance function of a structure can be represented as $Y = G(\boldsymbol{X})$, where $\boldsymbol{X} = (X_1, \ldots, X_d)$ denotes the vector of random input variables with the joint probability density function (PDF) $f_{\boldsymbol{X}}(\boldsymbol{x})$. The marginal PDF of X_i is denoted as $f_{X_i}(x_i)(i = 1, \ldots, d)$. When the input variables are independent with each other, $f_{\boldsymbol{X}}(\boldsymbol{x}) = \prod_{i=1}^{d} f_{X_i}(x_i)$. The failure probability $P(F)$ can be represented as

$$P(F) = P\{G(\boldsymbol{X}) \le 0\} = \int I_F(\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} = E_{\boldsymbol{X}}[I_F(\boldsymbol{x})], \tag{1}$$

where $I_F(\boldsymbol{x}) = 1$ if $G(\boldsymbol{x}) \le 0$ and $I_F(\boldsymbol{x}) = 0$ otherwise, $E[\cdot]$ is the expectation operator.

When input variable X_i is fixed at a certain value x_i, the conditional failure probability can be represented as

$$\begin{aligned} P(F|x_i) &= P\{G(\boldsymbol{X}) \le 0 | x_i\} = \int I_F(\boldsymbol{x}_{\sim i}, x_i) f_{\boldsymbol{X}_{\sim i}}(\boldsymbol{x}_{\sim i}) \mathrm{d}\boldsymbol{x}_{\sim i} \\ &= E_{\boldsymbol{X}_{\sim i}}[I_F(\boldsymbol{x}_{\sim i}, x_i)], \end{aligned} \tag{2}$$

where $\boldsymbol{X}_{\sim i} = (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_d)$ denotes all the input variables except X_i and $f_{\boldsymbol{X}_{\sim i}}(\boldsymbol{x}_{\sim i})$ is the joint PDF of $\boldsymbol{X}_{\sim i}$. The effect of the fixed value x_i of input variable X_i on the failure probability can be measured by the difference between $P(F)$ and $P(F|x_i)$, i.e.

$$s(x_i) = |P(F) - P(F|x_i)|. \tag{3}$$

It can be seen that $s(x_i)$ is a function only dependent on x_i. Since x_i is just a certain value of random input variable X_i with PDF $f_{X_i}(x_i)$, the average effect of input variable X_i on the failure probability can be represented by the expectation of $s(x_i)$, i.e.

$$\begin{aligned} E_{X_i}[s(x_i)] &= E_{X_i}|P(F) - P(F|x_i)| \\ &= \int_{X_i} |P(F) - P(F|x_i)| f_{X_i}(x_i) \mathrm{d}x_i. \end{aligned} \tag{4}$$

In order to obtain a sensitivity measure lying between 0 and 1, Cui et al. [36] proposed a normalized failure-probability-based global sensitivity measure η_i for input variable X_i, i.e.

$$\eta_i = \frac{1}{2} E_{X_i}|P(F) - P(F|x_i)| = \frac{1}{2} \int_{X_i} |P(F) - P(F|x_i)| f_{X_i}(x_i) \mathrm{d}x_i. \tag{5}$$

Eq. (5) shows that η_i measures the average change of the failure probability when input variable X_i is fixed. The sensitivity measure η_i has a similar form with the moment-independent sensitivity measure δ_i proposed by Borgonovo [32]. We can obtain δ_i by replacing the unconditional failure probability $P(F)$ and the conditional failure probability $P(F|x_i)$ in Eq. (5) with the unconditional PDF $f_Y(y)$ and conditional PDF $f_{Y|x_i}(y)$ of model output Y separately. The moment-independent sensitivity measure can reflect the average effect of input variable on the whole PDF of model output. The failure-probability-based global sensitivity measure can reflect the average effect of input variable on the failure probability. Since the failure probability is often related to the tail behavior of the distribution of model output, η_i can also reflect the effect of input variable X_i on the tail behavior of the distribution of model output. More details about η_i can be found in [36].

2.2. Estimation of the failure probability-probability-based global sensitivity measure

According to the definition in Eq. (5), it can be seen that the key of estimating η_i requires calculating the unconditional failure probability $P(F)$ and the conditional failure probability $P(F|x_i)$ with different values of X_i. Therefore, many methods for estimating failure probability can be utilized to estimate η_i. Since the MCS method is a widely used method for estimating failure probability, a traditional MCS method with double-loop sampling is introduced to estimate η_i in this subsection. Based on Eq. (5), the procedure for estimating η_i can be represented as follows.

(1) Generate a set of samples $(\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(N)})$ of input variables $\boldsymbol{X} = (X_1, X_2, \ldots, X_d)$ according to the corresponding joint PDF $f_{\boldsymbol{X}}(\boldsymbol{x})$. Then, estimate the failure probability $P(F)$ as

$$\hat{P}(F) = \frac{1}{N} \sum_{j=1}^{N} I_F(\boldsymbol{x}^{(j)}). \tag{6}$$

(2) Generate a set of samples $(x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(N_c)})$ of input variable X_i $(i = 1, \ldots, d)$ according to the corresponding PDF $f_{X_i}(x_i)$. For each sample $x_i^{(j)}$ $(j = 1, \ldots, N_c)$, generate a set of samples $(\boldsymbol{x}_{\sim i}^{(1)}, \boldsymbol{x}_{\sim i}^{(2)}, \ldots, \boldsymbol{x}_{\sim i}^{(N)})$ of the other input variables $\boldsymbol{X}_{\sim i} = (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_d)$ based on the corresponding joint

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