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Cooperative guidance with multiple constraints using convex optimization

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ABSTRACT

In the existing closed-loop cooperative guidance approaches for salvo attack of multiple missiles, the multiple constraints and time-variant velocity basically cannot be effectively considered. Therefore, two closed-loop cooperative guidance methods are developed in this paper, through employing the efficient convex optimization technique and receding horizon control (RHC) strategy. During each guidance cycle of RHC, the system coordination target is updated and then broadcasted to each missile as a constraint. Subsequently, the convex optimization technique is utilized to solve the multi-constraint optimal proportional guidance problem of each missile online to achieve the consensus on time-to-go among missiles. Simulation results show that for three cases with different conditions of velocity, the cooperative simultaneous attack under multiple constraints can be effectively carried out using each of the two proposed cooperative guidance laws, which verify their effectiveness and feasibility.

1. Introduction

Cooperative guidance for salvo attack of multiple missiles has been an active and attractive research topic because it may have better performance than the individual missile system in detecting the maneuvering targets, penetrating the defense systems, and surviving the threats [1–5]. In a salvo attack scenario, multiple missiles are required to hit the target simultaneously to introduce a many-to-one engagement situation for missile defense system. According to whether the missiles have dynamic information sharing during the course of guidance, the cooperative guidance can be generally categorized into open-loop cooperative guidance and closed-loop cooperative guidance [6]. For the former one, a common impact time is commanded to all members in advance before the attack, and thereafter each missile tries to arrive at the target on time independently. A closed form of impact time control guidance (ITCG) law was introduced based on the linear formulation to guide a group of missiles to simultaneously intercept a stationary target at a desirable time [7]. Later, an extension of the ITCG law to control both the impact time and the impact angle was developed by Lee et al. [8]. Meanwhile, a novel time-constrained guidance (TCG) law, which can control the flight time of missiles to a prescribed time, is designed by using the virtual leader scheme and stability method [9]. Clearly, during the open-loop cooperative guidance, the impact time must be preprogrammed manually into all missiles before they are launched and there is no communication and dynamic information sharing among the missiles. Therefore, it cannot be viewed as a genuine multi-missile cooperative attack.

For the closed-loop cooperative guidance, a two-level hierarchical cooperative guidance architecture with both centralized and distributed coordination algorithms for multi-missile attack was proposed by Zhao etc. based on the ITCG law, in which the desirable impact time is considered as the coordination variable and dynamically estimated during the course of guidance [10]. Based on the leader–follower strategy, a time-cooperative guidance architecture composed of individual guidance for each missile and coordination strategy of the whole system was proposed [11], and a time-cooperative control of multiple missiles was derived by adjusting the range-to-go and the heading error angle of the follower relative to the target to approach that of the leader [12]. A cooperative proportional navigation guidance law with a time-varying navigation gain for each missile was derived to achieve the salvo attack by decreasing the time-to-go variance cooperatively till the intercept [13]. A distributed guidance law for cooperative simultaneous attacks against a stationary target with multiple missiles

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Nomenclature

LOS	line-of-sight
PN	proportional navigation
RHC	receding horizon control
J	performance function
K	sequence number of guidance cycle
N	guidance coefficient
T_p	prediction horizon
T_c	receding horizon update time points
V	speed of the missile
a	acceleration of missile
r	LOS distance between missile and target
r_δ	pre-specified minimum LOS distance of all missiles
s	prediction horizon during guidance
t_{go}	time-to-go before arriving at target
t^*	designated system time-to-go
Δt	control update period
$\boldsymbol{x}_i$	state vector of the i-th missile
γ	flight-path angle
ε	look angle
λ	LOS angle
γ	flight path angle

was designed to achieve a consensus of the real times-to-go of missiles [14]. These preceding researches basically focus on obtaining the analytical cooperative guidance law and the key component of time-to-go (denoted as t_{go}) estimation for the missile is estimated based on the assumption that the velocity of each missile is constant. Therefore, the accuracy for these approaches may not be guaranteed for the scenarios with time-varying velocity [15,16]. To address this issue, an artificial neural network with extreme learning machine is introduced to estimate t_{go} by fitting the relationship between the t_{go} command and the local proportional guidance law in a distributed cooperative guidance strategy with consensus on the times-to-go of all missiles [15]. However, the learning speed of the artificial neural network may be slow, and thus this method cannot be used in real-time cooperative guidance.

Furthermore, in addition to the impact time, constraints like the impact angle, impact velocity, look angle and maximum lateral acceleration are usually being taken into account during the design of guidance law to improve the attack precision and performance, which clearly brings about great difficulties to the design of cooperative guidance law, especially those in the analytical forms. Recently, the guidance approaches considering multiple constraints for a single missile through computational methods, such as the online pseudo-spectral guidance [17,18], nonlinear programming guidance [19] and receding time domain optimization guidance [20,21], have gained much attention. With these computational-based approaches, the design of guidance law for a single missile is transcribed into an optimal control problem, which is then solved by numerical methods to obtain the optimal guidance law satisfying multiple constraints. It is very flexible and has become a research focus in recent years especially with the improvement of the computational capability of on-board computer. Considering its advantages in solving the nonlinear optimal control with multiple constraints, the Gaussian pseudo-spectral method has been employed to generate the cooperative guidance law numerically [22], but the impact time is pre-specified as done in the open-loop cooperative guidance. Meanwhile, the real-time capability and reliability of the Gaussian pseudo-spectral method cannot be ensured since the optimal control is transcribed into a nonlinear programming problem, which is inapplicable to online closed-loop guidance in practice [23]. A framework of a distributed closed-loop receding horizon control cooperative guidance scheme was developed, in which each interceptor missile is assigned its own finite-horizon optimal control problem solved by the particle swarm optimization (PSO) method aiming at minimizing the discrepancy of times-to-go among the missiles for salvo attack [24]. Similarly, the real-time capability and reliability of PSO cannot be ensured, inducing difficulty in closed-loop guidance. Meanwhile, the velocity of missile is considered to be constant, and then the time-to-go is estimated, which may be inaccurate to the cases with time-varying velocity. In recent years, due to their fast convergence speeds, the methods such as optimal guidance and predictive control, the convex optimization technique with great potential in real-time processing and capability of handling various constraints in the optimal control problems has been frequently discussed [25–27].

To address the time-variant velocity and multiple constraints during the cooperative guidance, two closed-loop cooperative guidance approaches respectively based on the leader–follower scheme [11,12] and the two-level hierarchical scheme [10] are developed, using the efficient convex optimization technique in conjunction with the receding horizon control (RHC) strategy. For both methods, the cooperative proportional navigation (CPN) scheme for simultaneous attack is employed, and the time-to-go is considered as the coordination variable that is dynamically estimated and shared among the missiles. During one guidance cycle of RHC, each missile is assigned its own finite-horizon optimal control problem with the time-varying navigation ratio as the control variable, which is solved online independently in a distributed manner by employing the highly efficient convex optimization technique to reduce the variance of times-to-go of missiles for salvo attack. Meanwhile, in order to avoid the errors of existing cooperative guidance methods in estimating the time-to-go t_{go}, the line-of-sight (LOS) distance between the missile and target is introduced as a new independent variable rather than the traditional time t to calculate the time-to-go.

The remainder of this article is organized as follows. In Section 2, the many-to-one guidance geometry with the time-varying proportional navigation gain is briefly reviewed, followed by a detailed description of the two proposed cooperative guidance frameworks in Section 3. In Section 4, numerical results of the proposed approaches are discussed in detail. Conclusions are finally drawn in Section 5.

2. Guidance geometry

Without loss of generality, the cooperative guidance for attacking a stationary target or low-speed target within the horizontal plane is considered, with the guidance geometry shown in Fig. 1. It is assumed that the missiles are launched simultaneously with synchronous time. In Fig. 1, V_i represents the speed of the i-th missile ($i = 1, 2, \ldots, N$), $t_{\text{go},i}$ the corresponding time-to-go before arriving at the target, λ_i and r_i are respectively the line-of-sight (LOS) angle and LOS range (i.e., range-to-go) between the i-th missile and the target, γ_i the flight-path angle and ε_i ($\varepsilon_i = \gamma_i - \lambda_i$) the look angle.

Meanwhile, the following assumptions are made in this paper as is commonly done when considering cooperative guidance problems for multiple missiles.

Assumption 1: the missile and target are treated as mass points in the planar plane.

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