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Feasible zone for planetary entry vehicles

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ABSTRACT

Understanding the coverage of ground track is conducive to evaluating an entry vehicle's maneuver capability, especially for an entry mission with waypoint constraint. This paper investigates a feasible zone which describes the lateral corridor of the entry vehicle's ground track. Any point inside the feasible zone is supposed to be reachable from the initial condition and controllable to the terminal condition. Analyses show that the zone boundary consists of two or three phases. For each phase, the zone boundary is generated by solving one or more trajectory optimization problems. The feasible zone can be employed to judge whether or not a waypoint is feasible to the vehicle. Moreover, to evaluate the feasible level of the waypoint, an evolved feasible zone with pass index is defined and calculated. The feasible zone is simulated for an Earth entry vehicle in various missions. Dispersions are considered in the simulation to analyze the influence of aerodynamic uncertainties.

1. Introduction

Atmospheric entry flight on either the Earth or Mars is highly constrained. The planetary entry vehicle needs to fly from a reasonable initial condition to a safe terminal condition with path constraints throughout the flight. Thus, to ensure a safe and reliable entry flight, it is of great importance to get in-depth knowledge of the flight conditions and constraints.

The initial and terminal entry conditions should be carefully selected according to the vehicle's control capability. Benito and Mease [1] investigated the controllable and reachable sets for the Mars entry. The controllable set is defined as a set of initial states that are controllable to a desired terminal condition. In a follow-up study [2], the controllable set was analyzed through a union set and a intersection set to discover the impact of model uncertainties. In Ref. [3], an orthogonal table was defined to evaluate the initial states and find an optimal initial condition for the entry flight. Trajectories starting from the initial states were generated using the Gauss pseudospectral method and evaluated by an integrated indicator. In contrast to the controllable set, the reachable set is defined as a set of terminal states that are reachable from a given initial condition [1]. The calculation of the full reachable set is time-consuming and sometimes unnecessary. Thus, the terminal condition is usually investigated through a landing footprint, which is a two-dimensional reachable set in the longitude vs. latitude plane, i.e., on the planet surface. See Refs. [4–8] for generation methods of the landing footprint.

Typical path constraints are given by limits of heating rate, dynamic pressure, and aerodynamic load [9,10]. In order to meet these constraints, entry corridors have been established in various planes, such as the drag-velocity corridor [9], the drag-energy corridor [11], and the altitude-velocity corridor [12]. In Ref. [13], the two-dimensional corridor is extended into a three-dimensional one which considers the flight-path angle. Apart from the path constraints, additional geographic constraints like waypoints and no-fly zones need to be considered in some penetration missions [14]. Waypoints, locations for scene matching or payload delivery, need to be passed through by the vehicle's ground track [15,16]. No-fly zones, such as the coverage of a missile defense system, are areas that the vehicle must avoid entering. In recent works, trajectory optimization/planning methods have been investigated for the entry flight under the waypoint and no-fly zone constraints [16–20]. To develop an autonomous guidance system, Refs. [21–25] designed onboard bank reversal strategies for the no-fly zone constraint. However, the current results are mainly derived for missions where the waypoint and the no-fly zone are predetermined. To the best of our knowledge, research on the waypoint selection and evaluation is still lacking. Actually, there may exist several candidate waypoints for an entry mission, and the designers concern how to select an appropriate one. The selection can be conducted by solving a waypoint-constrained trajectory optimization problem

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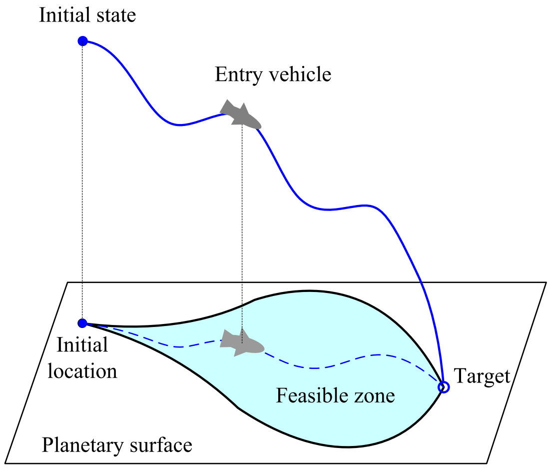

Fig. 1. Illustration of feasible zone for entry flight.

for each candidate. However, the computation would increase with the number of potential waypoints.

In this study, the feasible zone of the waypoint is investigated through the coverage of the entry ground track. Similarly to the controllable and reachable sets, the feasible set is defined as a set of states that are feasible to the vehicle when flying from a given initial condition to a desired terminal condition. The feasible set is projected onto the longitude vs. latitude plane, which generates a feasible zone. The feasible zone for an entry vehicle is illustrated in Fig. 1. All the locations in the zone are reachable from the initial location and controllable to the target (terminal location), or in other words, "passable" to the vehicle.

The entry trajectory and the ground track are determined by the bank angle profile with unknown magnitude and reversal time. A bank angle profile is feasible only if the corresponding trajectory meets the path and terminal constraints. Therefore, even for the nominal case, it is hard to generate a group of ground tracks that can totally cover the feasible zone, which means that the accurate feasible zone is hard to be directly obtained by Monte Carlo simulations. This work aims to calculate the feasible zone boundaries according to the maneuver characteristics of the vehicle. The left and right boundaries are each divided into two or three phases, and an optimization problem is formulated for each phase. Given the formulations, the zone boundaries can be generated using the trajectory optimization methods developed in previous works [16–20]. In this study, the hp-adaptive Gaussian quadrature collocation method embedded in the General Pseudospectral Optimal Control Software (GPOPS) is employed [26].

The feasible zone, which is developed for midcourse states, is an extension of previous works on the controllable set for initial states and the reachable set for terminal states. The feasible zone can be employed to evaluate the pass feasibility of any concerned location. A location is feasible to be a waypoint if and only if it is within the zone. The feasible zone, once obtained, does not need to be updated when employed to evaluate other locations. Since the feasible zone describes the coverage of the vehicle's ground track, it provides an aid for the detection and tracking. For instance, to achieve a better track of the entry vehicle, tracking ships can be arranged according to the feasible zone. Moreover, the feasible zone boundary also describes a lateral corridor of the vehicle's trajectory. When the vehicle is beyond the zone, it would have insufficient energy to reach the target.

This paper is organized as follows. Section 2 formulates the planetary entry problem. Section 3 defines the feasible zone and proposes a generation approach. In Section 4, the feasible zone is simulated in both the nominal and dispersed cases. Finally, Section 5 gives some conclusions.

2. Entry problem formulation

The state variables of an entry trajectory can be expressed as $\boldsymbol{X} = [r,\ \theta,\ \phi,\ v,\ \gamma,\ \psi]$, where r is the radial distance from the planet center to the vehicle, θ is the longitude, ϕ is the latitude, v is the velocity magnitude, γ is the flight-path angle, and ψ is the velocity heading angle. For an entry vehicle over a spherical planet, differential equations are given by [27]

$$\dot{r} = v \sin\gamma \tag{1}$$

$$\dot{\theta} = \frac{v\cos\gamma\sin\psi}{r\cos\phi} \tag{2}$$

$$\dot{\phi} = \frac{v\cos\gamma\cos\psi}{r} \tag{3}$$

$$\dot{v} = -D - g\sin\gamma + \omega^2 r\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\cos\psi) \tag{4}$$

$$\dot{\gamma} = \frac{1}{v}\left[L\cos\sigma - g\cos\gamma + \frac{v^2\cos\gamma}{r} + 2\omega v\cos\phi\sin\psi \right. \\ \left. + \omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\cos\psi)\right] \tag{5}$$

$$\dot{\psi} = \frac{1}{v}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{v^2\cos\gamma\sin\psi\tan\phi}{r} \right. \\ \left. - 2\omega v(\tan\gamma\cos\phi\cos\psi - \sin\phi) + \frac{\omega^2 r}{\cos\gamma}\sin\phi\cos\phi\sin\psi\right] \tag{6}$$

where g is the gravitational acceleration and ω is the self-rotation rate of the planet. The bank angle σ, the angle the lift vector is rotated around the velocity axis [1], is the main control variable of the entry trajectory. The lift acceleration L and the drag acceleration D are calculated by

$$\begin{cases} L = \dfrac{1}{2m}\rho v^2 S_A C_L(\alpha, Ma) \\ D = \dfrac{1}{2m}\rho v^2 S_A C_D(\alpha, Ma) \end{cases} \tag{7}$$

where m is the mass of the vehicle, and S_A is the reference area. The lift coefficient C_L and the drag coefficient C_D both depend on the angle of attack α and the Mach number Ma. In the entry trajectory planning, a specified angle of attack profile is usually utilized [6]. The atmospheric density ρ is modeled by an exponential function [28]:

$$\rho = \rho_0 e^{-h/H} \tag{8}$$

where ρ_0 is the density at sea level, h is the altitude, and H is a constant.

The initial and terminal conditions for the entry trajectory are expressed as

$$\boldsymbol{X}_0 = [r_0,\quad \theta_0,\quad \phi_0,\quad v_0,\quad \gamma_0,\quad \psi_0] \tag{9}$$

and

$$\boldsymbol{X}_f = [r_f,\quad \theta_f,\quad \phi_f,\quad v_f,\quad \gamma_f,\quad \psi_f] \tag{10}$$

The entry vehicle is expected to fly from the initial condition to the terminal condition with the path constraints

$$\boldsymbol{F}(\boldsymbol{X}) = \begin{bmatrix} K_Q\rho^{0.5}v^{3.15} - Q_{\max} \\ \sqrt{L^2 + D^2} - n_{\max} \\ 0.5\rho v^2 - q_{\max} \end{bmatrix} \le 0 \tag{11}$$

where K_Q is a constant. The maximum limits $Q_{\max}$, $n_{\max}$, and $q_{\max}$ are constraints for the heating rate, the aerodynamic load, and the dynamic pressure, respectively. The expression $\boldsymbol{F}(\boldsymbol{X}) \le 0$ represents that each of the three elements is no greater than zero.

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