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Generalized error analysis of analytical coarse alignment formulations for stationary SINS

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ABSTRACT

This communication presents additional results on a recently published work of the author [1], which addressed the stationary coarse alignment (CA) stage of strapdown inertial navigation systems (SINS). As main contribution of this communication, the error analysis proposed in [1] is extended, and novel general expressions for the SINS CA errors are derived, which are valid regardless of the inertial measurement unit (IMU) orientation, and present hence, greater practical applicability. The general error expressions prove to be similar to the simplified equations derived in [1], but with body frame coordinates replaced by navigation frame coordinates. Simulation results validates the adequacy of the outlined verifications, and are in agreement with experimental results found in the literature.

1. Introduction

Initial alignment is a very important procedure for strapdown inertial navigation systems (SINS). Basically, it consists of establishing the initial attitude matrix relating body and navigation frames, which will be used for initializing the numerical integration routines inside the inertial navigator, and for analytically resolving the angular rates and specific force accelerations measured by the inertial measurement unit (IMU) thereafter [2].

In strategic (military) applications, requirements for the SINS alignment include: autonomy, accuracy, rapidness and robustness [3]. The autonomy requirement, in specific, imposes the need of using intermediate-grade IMUs for the purpose of the alignment (angular rate sensors able to sense the Earth rotation rate). The accuracy and rapidness requirements, in turn, compel the alignment to be conducted into two separate phases: the coarse and the fine alignment [4,5]. The robustness, lastly, relates to the SINS capability of performing the alignment regardless of the generally "stationary" condition of the vehicle [6].

During the last decades, several stationary coarse self-alignment (CA) formulations for SINS have been proposed in the literature. The most traditional one is the three-axis attitude determination-based (TRIAD) method, which employs a set a three linearly independent vectors, formed by cross-product between the local gravity and Earth rate vectors, to establish the initial attitude matrix [7]. Another important CA method is the orthogonal (O)-TRIAD, which actually consists of a slight modification of the TRIAD method, originating reduced alignment errors and no orthogonality errors [8].

An alternative, and acknowledged, CA method for SINS is the attitude matrix decomposition-based alignment (ADIA) method. Differently from the TRIAD and O-TRIAD, the ADIA method establishes the vehicle initial attitude information through the decomposition of the local gravity vector in the inertial frame [9]. The approach used for solving this decomposition problem has originated different ADIA alternatives in the literature [10–13]. Despite being suitable for some particular cases, as the alignment of lower-grade IMUs subject to swaying conditions, these methods originate more corrupted and time-dependent attitude errors, which is a drawback.

In a recently published work of Silva et al. [1], the CA problem of SINS has been revisited and a modified version of the O-TRIAD has been proposed, namely, the orthonormal (ON)-TRIAD. As explained by the authors, the ON-TRIAD method is based on normality and orthogonality constraints existing in the attitude matrix supplied by the O-TRIAD and its corresponding Euler angle representation [14]. As an advantage of the ON-TRIAD, one completely eliminates the normality errors produced by the O-TRIAD, contributing, hence, to the reduction of the attitude, velocity and position error build-up during the posterior navigation stage.

In order to analytically validate the ON-TRIAD method, Silva et al. [1] presented a comprehensive error analysis, which considered, in addition to the inertial sensor uncertainties, the existence of uncertainties on the latitude and local gravity information. Despite the adequacy of the proposed error analysis, the latter has been restricted to the idealized case of body and navigation frames perfectly aligned, which is a very weak assumption in practical sit-

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uations, since it is rarely met. To validate the adequacy of the ON-TRIAD, regardless of the SINS orientation, only numerical results (both simulated and experimental) have been presented in [1].

In this communication hence, we extend the error analysis of Silva et al. [1] for the general case of body and navigation frames arbitrarily oriented, which presents much greater engineering applicability. As main contribution of the communication, we demonstrate that the resulting "general" error expressions are not messy and not readily amenable to physical interpretation, as originally stated in [7,8,1]. Instead, the general error expressions are similar to those presented in [1], but with body frame components replaced by components in the north, east and down directions (navigation frame coordinates). Moreover, the general error expressions are demonstrated to be identical to the accuracy that is often reported in fine alignment schemes, which is consistent with physical interpretation.

The remainder of this communication is structured as follows: Section 2 reviews the ON-TRIAD CA method for SINS and extends the error analysis of [1] to the general case of IMU arbitrarily oriented; Section 3, in sequence, provides results from simulated test; and Section 4, lastly, introduces the conclusions and final considerations.

2. Coarse alignment and generalized error analysis

As mentioned in Section 1, the recently proposed orthonormal (ON)-TRIAD coarse alignment (CA) method for SINS derives from normality and orthogonality properties existing between the attitude matrix supplied by the orthogonal (O)-TRIAD and its corresponding Euler angle representation. As demonstrated by Silva et al. [1], the initial attitude matrix attainable with the ON-TRIAD corresponds to

$$C_b^l = \begin{bmatrix} \dfrac{\mu}{\nu} & \dfrac{\alpha a_x a_y - \epsilon g_P^2 a_z}{\alpha\nu\xi g_P a_z} & \dfrac{\alpha a_x a_z + \epsilon g_P^2 a_y}{\alpha\nu\xi g_P a_z} \\ \dfrac{\epsilon\mu g_P}{\alpha\nu} & -\dfrac{\zeta(a_y^2+a_z^2)}{\alpha\nu\xi a_z} & -\dfrac{\vartheta(a_y^2+a_z^2)}{\alpha\nu\xi a_z} \\ -\dfrac{a_x}{g_P} & \dfrac{\mu a_y}{\xi a_z} & \dfrac{\mu}{\xi} \end{bmatrix} \tag{1}$$

with $a_z \neq 0$ and

$$\epsilon = a_z\omega_y - a_y\omega_z \tag{2}$$

$$\zeta = a_x\omega_z - a_z\omega_x \tag{3}$$

$$\vartheta = a_y\omega_x - a_x\omega_y \tag{4}$$

$$\alpha = \vartheta a_y - \zeta a_z \tag{5}$$

$$\mu = \sqrt{1 - \frac{a_x^2}{g_P^2}} \tag{6}$$

$$\nu = \mathrm{sign}(\omega_x)\sqrt{1 + \frac{\epsilon^2 g_P^2}{\alpha^2}} \tag{7}$$

$$\xi = \sqrt{1 + \frac{a_y^2}{a_z^2}} \tag{8}$$

where g_P is the magnitude of the local gravity vector, and a_x, a_y, a_z and ω_x, ω_y, ω_z are the specific force and angular rate vectors components in body-frame coordinates, respectively.

As analyzed by Silva et al. [1], the ON-TRIAD is superior to the O-TRIAD (and to the remaining TRIAD and ADIA formulations mentioned in Section 1) because it guarantees the normality and orthogonality properties of the ideal C_b^l attitude direct cosine matrix (DCM), by additionally dissociating the east alignment error from the angular rate sensor uncertainties [1].

In order to validate the ON-TRIAD method, Silva et al. [1] proposed a comprehensive error analysis, which considered, in addition to the inertial sensor uncertainties, latitude and local gravity uncertainties. Silva's error analysis basically consisted of analytically investigating the properties of the computed (and corrupted) $\hat{C}_b^l$ matrix, which has been equated as

$$\hat{C}_b^l = C_b^l + \delta C_b^l = (I + E)C_b^l \tag{9}$$

with

$$E = \delta C_b^l (C_b^l)^T \tag{10}$$

where I is the identity matrix and δC_b^l is the matrix containing $\hat{C}_b^l$ errors.

As explained by Savage [15], the E matrix can also be seen as an association of two complementary error matrices,

$$E = E_s + E_{ss} \tag{11}$$

defined as

$$E_s = \frac{E + E^T}{2} = \begin{bmatrix} \eta_N & o_D & o_E \\ o_D & \eta_E & o_N \\ o_E & o_N & \eta_D \end{bmatrix} \tag{12}$$

$$E_{ss} = \frac{E - E^T}{2} = \begin{bmatrix} 0 & \varphi_D & -\varphi_E \\ -\varphi_D & 0 & \varphi_N \\ \varphi_E & -\varphi_N & 0 \end{bmatrix} \tag{13}$$

both of which representing the $\boldsymbol{\eta}$ and $\boldsymbol{o}$ attitude matrix normality and orthogonality error vectors, and the $\boldsymbol{\varphi}$ attitude matrix alignment error vector, respectively.

In order to develop their error analysis, Silva et al. [1] expanded (1) as

$$\delta C_b^l = \frac{\partial C_b^l}{\partial a_x}\delta a_x + \frac{\partial C_b^l}{\partial a_y}\delta a_y + \frac{\partial C_b^l}{\partial a_z}\delta a_z + \frac{\partial C_b^l}{\partial \omega_x}\delta\omega_x + \frac{\partial C_b^l}{\partial \omega_y}\delta\omega_y + \frac{\partial C_b^l}{\partial \omega_z}\delta\omega_z + \frac{\partial C_b^l}{\partial L}\delta L + \frac{\partial C_b^l}{\partial g_P}\delta g_P \tag{14}$$

where δ is the generic representation for error quantities.

Resorting to symbolic computational aid, these authors analytically solved (14) and the result, substituted in (10) (jointly with (1)), then in (12) and (13), was used to produce sensitivity expressions for the attitude matrix normality, orthogonality and alignment errors. Despite its adequacy, a very weak assumption has been considered in [1], namely: the ideal scenario of body and navigation frames perfectly aligned. As this assumption is generally not met in practical situations, the error expressions derived in [1] are restrictive, and possess reduced practical applicability. When the related literature is consulted hereupon, we verify that the currently available works also fail to provide general error expressions for the SINS CA problem [7,8,16]. The authors of these works claim, essentially, that the resulting expressions would be quite messy and not readily amenable to physical interpretation.

In order to make Silva's error analysis more generic and coherent to practical implementations (body and navigation frames arbitrarily oriented), let us, initially, consider the idealized (error-free) body-frame[1] representation of the $\boldsymbol{a}_{SF}$ specific force vector, measured by the accelerometers, and the $\boldsymbol{\omega}_{ib}$ angular rate vector, measured by the angular rate sensors, i.e.,

[1] In this communication, the body frame is represented by the index b, and is defined with the x_b, y_b and z_b axes pointing forward, to the right-hand side and downward, all with respect to the vehicle which the system is mounted on.