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Dynamics of flexible multibody systems with variable-speed control moment gyroscopes

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A B S T R A C T

This paper presents a generic global matrix formulation for the dynamics of flexible multibody systems with variable-speed control moment gyroscopes (VSCMGs). The flexible bodies are assumed to exhibit only small deformation, and they are connected in a tree topology by hinges permitting large rotation and translation. A cluster of VSCMGs is mounted on each body for actuation; it is assumed that the VSCMGs are statically and dynamically balanced, and their rotors are axisymmetric. A minimum set of dynamic equations are derived systematically via a mixed use of Kane's method and Newton–Euler equations. The parameters of each flexible body are augmented to take into account the inertias of the attached VSCMGs. Moreover, a skew-symmetric gyroscopic matrix and three control-input-mapping matrices are defined to represent the passive and the active gyroscopic torques of the VSCMGs in a global matrix manner. Three examples are given to show the usefulness, versatility, and correctness of the proposed formulation. As an additional contribution, also presented are linear dynamic equations for a spacecraft with flexible appendages and embedded VSCMGs.

1. Introduction

Momentum exchange devices, such as control moment gyroscopes (CMGs) and momentum wheels (MWs), are widely used in spacecraft attitude control systems [1]. Moreover, they have been used innovatively in the design of many next-generation space systems [2]. These novel systems, which will be introduced in the next two paragraphs, are special cases of flexible multibody systems with variable-speed CMGs (VSCMGs). The main contribution of this paper is a generic global matrix dynamics formulation for this kind of systems. It extends the state-of-art global matrix formulations for ordinary flexible multibody systems by including the effects of VSCMGs using global matrices.

In the operation of space structures, vibration control is needed to meet the extremely accurate pointing and shape requirements [3,4]. Single-gimbal CMGs (SGCMGs) were proposed as "gyrodampers" for vibration suppression in Ref. [5]. D'Eleuterio and Hughes proposed to embed a cloud of MWs and/or CMGs in a large space structure, and represent them by a continuous distributed "gyricity" [6]. The mere presence of the "passive" gyricity can stabilize the shape of the "gyroelastic" spacecraft. Furthermore, by "active" steering, vibration can be suppressed [7]. While the gyricity concept is elegant and applies well in theoretical developments, practical applications need formulations with detailed description of the momentum exchange devices. Recent studies on gyroelastic spacecrafts with detailed discrete description of SGCMGs can be found in Refs. [8–14].

In the control of space multibody systems, momentum exchange devices can be used to steer the relative motion between different bodies as well as the attitude motion of the whole system. Scissored pairs of SGCMGs were used to control rigid space robots in Refs. [15,16], with which the disturbance torque exerted by the manipulator on the spacecraft bus was reduced [17]. The design and energetics of CMG-actuated space robots were considered in Refs. [18,19], the energy-optimal maneuver problem was studied in Refs. [20,21], and a momentum equalization controller was developed in Ref. [22]. The CMG-actuation concept was pushed further by using a cluster of SGCMGs on each manipulator link in Refs. [23,24]. In Ref. [25], another generalization was made by using VSCMGs to actuate a dual-body spacecraft. The use of VSCMGs in maneuvering and vibration control of flexible space robots were studied in Refs. [26,27].

With the growing interests in novel applications of the momentum exchange devices, new system configurations will be proposed. A generic multibody dynamic formulation will make the dynamic analysis and controller design for these new systems more efficient with minimized equation-derivation effort, compared with deriving a set of customized equations for each system config-

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uration. The main motivation of this paper is to obtain such a formulation that handles different system topologies automatically, and that models the momentum exchange devices uniformly as VSCMGs, which can be degenerated to MWs or SGCMGs. Besides, despite the different configurations of the previously mentioned systems, similarities can be found in their dynamic equations, thus it is helpful to explore the common structure of these equations through a systematic derivation of the generic formulation.

Another major motivation of this work is to develop the generic formulation in terms of global matrices. The global matrices in dynamic equations concisely represent the coupling among different degrees of freedom, thus are used extensively in dynamic analysis and controller design. Examples of global matrices are mass and stiffness matrices of a structure, and their state-dependent counterparts in the multibody context [28–32]. In Ref. [6], a skew-symmetric gyroscopic matrix was defined to represent the effect of the passive gyroscopic torques of spinning rotors on space structures. The presence of this matrix results in “gyroelastic modes” of the gyroelastic spacecrafts rather than ordinary elastic modes. In Refs. [8,9,12,13], the active gyroscopic torques of SGCMGs on space structures were mapped into generalized forces using a control-input-mapping matrix, and the matrix was used to design and analyze vibration suppression controllers. These two matrices are also found in the optimal control law design in Ref. [7], the controllability and observability analysis in Ref. [33], and the gyricity distribution optimization in Refs. [11,14,34]. Given the importance of these two constant gyroscopic-torques-associated global matrices, their state-dependent multibody counterparts must be incorporated in the generic formulation.

A recursive dynamics algorithm was introduced in Ref. [2] for flexible multibody systems with VSCMGs. This algorithm extended the well-established recursive dynamic formulations for ordinary multibody systems [35–39] by taking into consideration the inertial properties and the detailed dynamics of the VSCMGs. It is very efficient for simulation, especially for systems with a large number of degrees of freedom, because the number of arithmetic operations for each integration step is $O(n)$, where n is the system's number of degrees of freedom. However, recursive dynamic algorithms do not readily provide the global matrices, because they solve for the accelerations one by one recursively.

Alternatively, global matrices are formulated explicitly in the standard global matrix formulations for flexible multibody systems in chain [28] and tree [29–32] topologies. However, these works did not take VSCMGs into account. A global matrix formulation with the effect of VSCMGs was given in Ref. [26]. It was found that the inertia of the VSCMGs contributes to the global matrices of generalized inertial forces defined in the standard formulations [28–32]. However, the system topology in that work was restricted to be a chain, and more importantly, the gyroscopic torques of the VSCMGs were not written in terms of global matrices. To conclude, it is needed to systematically develop a generic and complete global matrix formulation for flexible multibody systems with VSCMGs.

In this paper, we focus on systems in tree topologies; because many systems of engineering interest have a tree topology, and closed-loop constraints can be handled with minor changes to the proposed formulation by incorporating techniques such as new form of Kane's equation [32]. In Section 2, a detailed system description is provided, and key kinematic relations are developed. Based on these relations, a mixed approach is adopted to derive the global matrix formulation. Dynamic equations for the flexible bodies are first derived via Kane's method in Section 3, and then the effects of the VSCMGs are incorporated using Newton–Euler equations in Section 4. Particular care is taken in writing the effects of the VSCMGs using global matrices, and the structures of these matrices are fully explored. The usefulness and versatility of the proposed formulation are demonstrated through three application examples in Section 5. In the first example, the usefulness of the global matrices, especially those associated with the VSCMGs, is shown through the gyroelastic modal analysis and the vibration-suppression controller synthesis for a hinge-connected space structure equipped with VSCMGs. In the second example, versatility of the proposed formulation is shown in the dynamic simulation of a VSCMG-actuated flexible space robot in a representative tree topology with various kinds of hinges. The simulation results are compared against those of the well-established recursive formulation [2] introduced previously to verify the proposed formulation. Although one of our motivations is to minimize the derivation labor through a generic formulation, the proposed formulation can also be used to manually derive customized dynamic equations for systems in simple yet important topologies. In the third example, linear dynamic equations for a spacecraft with flexible appendages and VSCMGs are derived systematically with the proposed formulation, which are also a contribution to the literature.

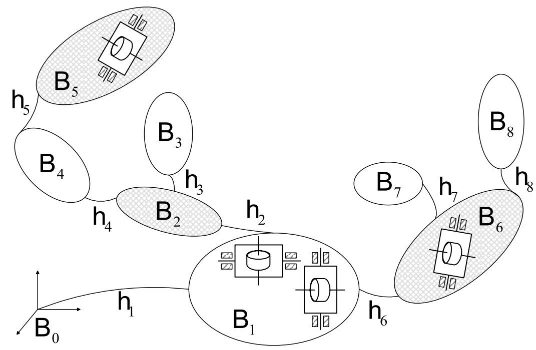

Fig. 1. Flexible multibody system in a tree topology with VSCMGs.

2. System description and kinematics

2.1. Topological array and motion variables

An example tree-topology multibody system equipped with VSCMGs is shown in Fig. 1. The inertially fixed reference body is denoted by B_0, and the bodies in the system are numbered from B_1 to B_N, where N is the number of bodies. We use $c(j)$ to represent the index of the inboard adjacent body of B_j, which is in the path from B_j to B_0, and assume that $c(j) < j$ is satisfied when numbering the bodies. The array c is called the topological array of the system [30], which fully describes the system topology. c of the example system is

j	1	2	3	4	5	6	7	8
$c(j)$	0	1	2	2	4	1	6	6

The inboard adjacent hinge of B_j is denoted by h_j.

For simple notation in the derivation, we assume that all the bodies are flexible, that all the hinges are movable, and that each body is equipped with a cluster of VSCMGs. Degenerated cases such as rigid bodies, fixed hinges, and bodies without VSCMGs can be incorporated by omitting the corresponding terms in the generic equations.

To describe a generic hinge h_j, we define two dextral orthonormal frames, namely, F_j fixed to a node O_j on B_j, and F_{j^-} fixed to a node O_{j^-} on $\mathsf{B}_{c(j)}$, as shown in Fig. 2. h_j enforces constraints on the relative motion of F_j with respect to F_{j^-}. Because most types of hinges in engineering applications enforce time invariant and holonomic constraints, such as prismatic, revolute, spherical, and fictitious (free) hinges, we assume that