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Gas kinetic scheme for turbulence simulation

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ABSTRACT

The extended gas-kinetic scheme (GKS) for turbulence simulations is developed based on the generalized BGK equation with the effective relaxation time. This relaxation time can be computed from the turbulent viscosity, through which turbulence models can be directly combined. For engineering low-cost simulations of high Reynolds number flows, common-used RANS models are applied, while the LES and hybrid RANS/LES (DES and IDDES) models as well as the minimized dispersion and controllable dissipation (MDCD) reconstruction are adopted in high fidelity turbulence simulations. In addition, the turbulent transport equations are solved in a strongly coupled way by using GKS with scalar transport. The extended GKS is applied in typical turbulent flow predictions including the RANS simulation of hypersonic compression ramp flow and the detailed simulation of multiscale turbulent structures in low-speed cylinder flow with hybrid models. The predicted results agree well with existing experimental measurements and numerical studies, which shows the good accuracy, resolution and robustness of the extended GKS and reveals the wide prospects in turbulence simulations on different model scales, including the multiscale models such as hybrid RANS/LES methods. Furthermore, the multiscale turbulence simulation methods are worthy of further studies based on the generalized BGK model.

1. Introduction

The gas-kinetic scheme (GKS or GKS-NS) is a new CFD method based on the approximate solution of mesoscopic BGK equation on Navier–Stokes (NS) level [1]. In this scheme, the viscous and inviscid transports are coupled automatically and with consistent inherent dissipation mechanism, which guarantees its good performances in various flow problems, such as hypersonic viscous flows [2,3], magnetohydrodynamics [4], as well as the high accuracy and high resolution schemes for flow fine simulations [5–7]. The most existing applications focus on laminar flows so far.

Turbulence is a typical flow problem in engineering applications. Due to the multiscale features, it is a challenge to balance the accuracy requirements and computational costs [8,9] in numerical simulations, especially for high-Reynolds-number flows. Among turbulence models or simulation methods, the direct numerical simulation (DNS) solves the NS equations on grid cell size near the smallest (dissipation) scale. DNS method can capture fluctuations on all the scales, but the computational cost is too high for engineering high-Reynolds-number flow simulations. On the contrary, with the help of turbulence models, the Reynolds averaged Navier–Stokes (RANS) equations can be cheaply solved with coarse grid only to capture the very large (integral) scale structures. Different from DNS and RANS, large eddy simulation (LES) [10] handles the filtered NS equations, and is closed by additional sub-grid models based on the filtering scale or grid cell size. Thus LES can be regarded as a multiscale method. To further reduce the grid cost of LES in near wall regions where turbulent fluctuations are strong anisotropic and typically small scale, the hybrid RANS/LES method is proposed and become the hot topic in turbulence high fidelity simulations [11]. The hybrid method is also the multiscale or multilevel method [12], which can keep good balance between flow resolution accuracy and computational cost.

GKS can be directly applied to DNS of low-Reynolds-number turbulent flows, such as the DNS of mixing layer [13,14], the homogeneous turbulence [15,16] and the channel turbulent flow [17]. For high-Reynolds-number turbulence, the extended BGK equation with the effective relaxation time τ_e can be used to describe the relaxation of turbulent fluctuations [18]. τ_e can be determined by traditional turbulence models. The corresponding scheme is the extended GKS, and will be introduced in details in the following. In our previous studies, typical turbulence models such as Baldwin–Lomax (BL), k–ω SST and RNG k–ε models were implemented in GKS and showed good performances [19,20]. Similar conclusions are also got in other studies [21,22]. In recent research [23], k–ω model is considered in GKS via τ_e in renormalized form [18,24], and the turbulent 'rarefaction' effect is proposed and discussed. In

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turbulence fine simulations, LES simulation with GKS has also been conducted in complex flows around a car side mirror [25].

Despite many existing studies on the extended GKS, there are still some problems worthy of further investigations. The first one is the influences of numerical schemes and turbulence models. The self-adapting numerical dissipation [1] in the extended GKS is of more interest which comes from the cross-scale evolving solution of the extended BGK equation. The performance in hypersonic turbulent flows also requires evaluation. The second one is the capacity of the extended GKS in high fidelity simulations with multiscale models such as the hybrid RANS/LES methods. This work is expected to be a basis for further researches of turbulence simulating and modeling based on gas kinetic theory.

The rest of this paper is arranged as follows. The construction of GKS is briefly introduced in section 2. The construction and analysis of the extended GKS for turbulence simulations are in section 3, while followed by the numerical simulations and discussions in section 4. And the final section is the conclusions.

2. Gas-kinetic scheme

GKS which describes macroscopic flows from the mesoscopic BGK-Boltzmann equation is established by Xu et al. [1,3]. The scheme is briefly introduced in follows.

The BGK equation is,

$$\frac{\partial f}{\partial t}+\mathbf{u}\cdot\frac{\partial f}{\partial \mathbf{x}}=\frac{g-f}{\tau}, \tag{1}$$

where τ is the relaxation time. f is the gas distribution function and g is the equilibrium state. f and g are both functions in the high-dimensional phase space $(\mathbf{x}, t, \mathbf{u}, \xi)$ where ξ is the internal degree of freedom. And g is the Maxwell distribution,

$$g=\rho\left(\frac{\lambda}{\pi}\right)^{(K+3)/2}\mathrm{e}^{-\lambda(|\mathbf{u}-\mathbf{U}|^2+\xi^2)}. \tag{2}$$

Here $\lambda=\rho/(2p)$, $\xi^2=\xi_1^2+\xi_2^2+\ldots+\xi_K^2$. K is the total number of internal degrees of freedom, with value $K=2$ for the diatomic gas molecule in three-dimensional (3D) flows.

If f is known, the macroscopic conserved quantities and fluxes at the cell interface in the finite volume scheme can be got,

$$\begin{aligned}\mathbf{Q}&=(\rho,\rho\mathbf{U},\rho E)^T=\int\mathbf{\Psi}f\mathrm{d}\Xi,\\ \mathbf{F}_m&=\int u_m\mathbf{\Psi}f\mathrm{d}\Xi,\quad m=1,2,3.\end{aligned} \tag{3}$$

During the particle collision processes, f and g satisfy the conservation constraint,

$$\int(g-f)\mathbf{\Psi}\mathrm{d}\Xi=\mathbf{0}, \tag{4}$$

where $\mathrm{d}\Xi=\mathrm{d}u_1\mathrm{d}u_2\mathrm{d}u_3\mathrm{d}\xi_1\mathrm{d}\xi_2\ldots\mathrm{d}\xi_K$, $\mathbf{\Psi}=\left(1,\mathbf{u},\frac{1}{2}(|\mathbf{u}-\mathbf{U}|^2+\xi^2)\right)^T$.

BGK equation Eq. (1) has a general solution,

$$\begin{aligned}&f(\mathbf{x},t,\mathbf{u},\xi)\\&=\frac{1}{\tau}\int_0^t g(\mathbf{x}',t',\mathbf{u},\xi)\mathrm{e}^{-(t-t')/\tau}\mathrm{d}t'+\mathrm{e}^{-t/\tau}f_0(\mathbf{x}-\mathbf{u}t,\mathbf{u},\xi).\end{aligned} \tag{5}$$

Here f_0 is the initial distribution function at $t=0$, in which $\mathbf{x}'=\mathbf{x}-\mathbf{u}(t-t')$ is the trajectory of particle motions. The characteristic scale of f_0 is the kinetic scale, while the characteristic scale of g is the hydrodynamic scale. Thus, the general solution Eq. (5) naturally contains the multiscale transition between these two extreme scales, and the transition process is controlled by $\Delta t/\tau$. When simulating flows on NS level, f_0 can be constructed by the first-order Chapman–Enskog expansion instead of being directly discretized in the high dimensional phase space, while the Taylor expansion is adopted to achieve the target accuracy order. Therefore, huge amounts of computational costs can be avoided.

In the second-order accurate GKS, the first order Taylor expansion is adopted, and then f_0 and g in Eq. (5) are constructed as, (taking the normal direction x_1 of the cell interface as an example),

$$\begin{aligned}f_0(\mathbf{x},\mathbf{u},\xi)&=\left(1+a_m^l x_m-\tau(a_m^l u_m+A^l)\right)(1-\mathrm{H}[x_1])g^l\\&\quad+\left(1+a_m^r x_m-\tau(a_m^r u_m+A^r)\right)\mathrm{H}[x_1]g^r,\\ g(\mathbf{x},t,\mathbf{u},\xi)&=(1+(1-\mathrm{H}[x_1])\bar{a}_m^l x_m+\mathrm{H}[x_1]\bar{a}_m^r x_m+\bar{A}t)g_0.\end{aligned} \tag{6}$$

The coefficients $a_m^{l,r}$, $\bar{a}_m^{l,r}$, $A^{l,r}$ and $\bar{A}$ come from the derivatives of Maxwellian distribution, and can be computed by the gradients of macroscopic conserved quantities. The superscripts l, r represent the quantities on the left or the right side of the cell interface, respectively. $\mathrm{H}[x]$ is the Heviside function to account for the discontinuity at a cell interface. So the time evolving distribution function f at a cell interface ($\mathbf{x}=\mathbf{0}$) can be explicitly expressed in terms of local Maxwellian distributions,

$$\begin{aligned}f(\mathbf{0},t,\mathbf{u},\xi)&=(1-C_0)g_0+(t-\tau+C_1)\bar{A}g_0\\&\quad+(-\tau+C_1+C_2)(\bar{a}_m^l u_m\mathrm{H}[u_1]\\&\quad+\bar{a}_m^r u_m(1-\mathrm{H}[u_1]))g_0\\&\quad+\left(C_0-(C_1+C_2)a_m^l u_m-C_1A^l\right)\mathrm{H}[u_1]g^l\\&\quad+\left(C_0-(C_1+C_2)a_m^r u_m-C_1A^r\right)(1-\mathrm{H}[u_1])g^r,\end{aligned} \tag{7}$$

where $C_0=\mathrm{e}^{-t/\tau}$, $C_1=\tau C_0$ and $C_2=tC_0$. With f, the macroscopic numerical fluxes can be calculated according to Eq. (3), and the macroscopic conserved quantities are updated through the finite volume method. Besides, the second-order accuracy in both time and space can be achieved with a single step. Meanwhile, the distribution function f in Eq. (7) is a combination of Maxwellian function through which the integration to compute macroscopic quantities is very simple. So the computational cost is competitive with traditional schemes directly based on macroscopic governing equations. The usual reconstruction techniques can be directly applied to obtain the macroscopic quantities and their slopes at the cell interface. More details of GKS can be found in the literature [1,3,5,26] and will not be repeated here.

It should be noted that in the above distribution function, the particle movements in both the normal and tangential directions of the cell interface are taken into account, thus the resulted GKS is truly multidimensional which is difficult for traditional CFD schemes based on macroscopic Euler equations. If neglect the tangential coefficients, the directional splitting scheme can be recovered. Besides, the viscous effect is controlled by the collision time τ which is computed by

$$\tau=\frac{\mu}{p}+C\Delta t\frac{|p^l-p^r|}{|p^l+p^r|}. \tag{8}$$

Here μ is the molecular viscosity, while $p^{l,r}$ is the reconstructed pressure at the corresponding side of a cell interface. The constant $C=1$ is simply set and the time step Δt is constrained by the Courant–Friedrichs–Lewy condition. The last term in the above equation takes into account the effect of the discontinuity at a cell interface. The free transport of particle in f_0 leads to the upwind characteristics of the corresponding scheme, such as the kinetic flux vector splitting method (KFVS) [27]. The equilibrium state g corresponds to a central scheme. It should be noted that if f_0 and