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Buckling analysis of two-directionally porous beam

Haishan Tang, Li Li, Yujin Hu *

State Key Lab of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

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ABSTRACT

In this paper, buckling analysis of two-directionally porous beam is conducted. Based on the available results of Young's modulus and mass density via Gaussian random field theory, a new two-directionally porous beam model is developed. With the help of Euler–Bernoulli beam theory and minimum total potential energy principle, the equilibrium equations for nonlinear and linear buckling are derived. The numerical solutions of critical buckling loads for different porosity distribution patterns can be obtained by generalized differential quadrature method. The final numerical results exhibit that more porosities near the middle surface or the two edges of beam can lead to a larger critical buckling load when the same total volume fraction of porosity is in different porosity distribution patterns. The effect of porosity distribution in thickness direction is more dominated on the critical buckling load than that of the axial porosity distribution. Moreover, the critical buckling load becomes more sensitive to aspect ratio of beam and total volume fraction of the porosity when increasing mode number. The critical buckling load of two-directionally porous beam depends not only on bending coefficient (like the one-directionally porous beam), but also on first and second derivatives of the bending coefficient.

1. Introduction

Porosity, which is often inevitable in the actual manufacturing process of many materials, or formed intentionally to meet practical performance requirements in engineering. Compared with conventional materials, porous material has a strong designability, and it can possess fine-tuned porosity-dependent properties by tailoring its architecture. Up to now, the porous material has been playing an important role in a wide range of applications including topology optimization, micro-electromechanical systems, biotechnology, chemical techniques and so on. For example, nanoporous ceramic can be used to fabricate the gas-separating asymmetric ceramic membranes [1]. Porous sorbent material can remove the organics, particularly, it has a great potential to be utilized in worldwide oil governance such as oil upgrading and pollutants removal [2]. Porous zeolites have specific chemical properties to apply in size- and shape-selective catalysis and separation [3]. Kim et al. [4] investigated the noise absorption properties of porous materials by utilizing the Delany–Bazley empirical material model. It has been reported in [5–7] that porosity deemed as gas phase can effectively tailor material dielectric properties. Also, some porous materials can be applied in artificial organs [8–10], energy absorption devices [11–13], electromagnetic equipments [14,15], structural health monitoring [16], and displacement amplifier [17] due to their distinctive mechanical properties, especially the negative Poisson's ratio property.

Buckling instability is one of the most important problems encountered in engineering, which occurs when the external load exceeds the critical buckling load. In recent years, there has been an unprecedented upswing on the buckling analysis of beams, planes and shells [18–31]. On the one hand, porous material can tailor the static and dynamic behaviors of structures by different porosity distributions. On the other hand, the inappropriate porosity design, especially in the case of stress fatigue, may cause serious engineering problems, such as crack germination and growth [32–34]. Therefore, it is highly necessary to carry out the static and dynamic analyses of different porous structures exactly. Magnucki et al. [35] derived the analytical solution for the critical load of a considered beam made of isotropic one-directionally porous material. Magnucka-Blandzi et al. [36] conducted the buckling and bending analyses of a circular porous plate acted by buckling force and uniformly distributed load. Overvelde et al. [37] investigated the effect of porosity shape on the buckling behavior of two-dimensional periodic porous structures and found the porosity shape can be helpful for controlling some characteristics of soft porous systems. The buckling analysis of functionally graded porous circular plate was studied in Ref. [38], and the effects

* Corresponding author.
E-mail address: yjhu@mail.hust.edu.cn (Y. Hu).

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Nomenclature

"O", "Ō", "X", "X̄"	Four kinds of porosity distribution patterns	E_0	Young's modulus of material without porosity
x, y, z	different coordinates along length, width, thickness directions of beam	u, w	axial and transverse displacements for the mid axis
L	length of beam	P	axial compressing force
h	thickness of beam	P_{cr}	dimensional critical buckling load
b	width of beam	$\bar{P}_{cr}$	dimensionless critical buckling load
α	total volume fraction of porosity	I	second moment of cross section
V_p	volume fraction of porosity for different locations	A_{xx}	extensional coefficient
ρ	mass density of porous beam	B_{xx}	extensional-bending coupling coefficient
E	Young's modulus of porous beam	D_{xx}	bending coefficient
ρ_0	mass density of material without porosity	N	resultant force for the beam's cross section
		M	resultant moment for the beam's cross section

of porosity, thickness of plate, and different boundary conditions were presented.

As for the above-mentioned buckling analysis of porous structures, it is found that their porous models are one-directional, that is, the Young's modulus and the mass density of them change only in the thickness direction [39–41]. However, in fact, the volume fraction of porosity can vary with the three-dimensional (3D) position coordinates in most porous materials, i.e., the Young's modulus and the mass density of porous models may vary along different directions. Therefore, the one-directionally porous models widely used in the static and dynamic analyses have some limitations. It is urgent to propose a more practical porous model for our buckling analysis. First of all, the assessment of Young's modulus and mass density of the porous beam is an extraordinary basic and critical work. Some evidence in Refs. [42–47] has shown that the location, shape, size, volume fraction of the porosity exert significant influence on the Young's modulus and mass density. Moreover, the experimental data in Ref. [48] indicates that the Young's modulus of the porous material is also related to its mass density. According to the Voronoi tessellations [49] and Gaussian random fields theories [50,51], Roberts et al. [46,47] proposed general theoretical results of Young's modulus and mass density for random 3D closed-cell and open-cell porous materials by finite element method (FEM). These results of Young's modulus and mass density are highly consistent with the experimental data and very successful when they are exploited in porous structural modeling [52,53,40,54,55]. Therefore, the available results of Young's modulus and mass density via Gaussian random field theory can be utilized to develop a new two-directionally porous beam model.

In this study, the new two-directionally porous beam model will be proposed and employed in buckling analysis of two-directionally porous beam. At first, the two-directionally porous beam model considering several typical porosity distribution patterns will be presented in Section 2. Next, the equilibrium equations for nonlinear and linear buckling of the two-directionally porous beam will be deduced by Euler–Bernoulli beam theory and minimum total potential energy principle in Section 3. Whereafter, the generalized differential quadrature method can be used to solve the equilibrium equations for buckling of the two-directionally porous beam in Section 4. At last, some meaningful numerical results are discussed and presented in Section 5.

2. Modeling of two-directionally porous beam model

In this section, a new two-directionally porous beam model will be introduced. As shown in Fig. 1, we consider a beam model with length L, thickness h and width b, and the beam is made of two-directionally porous material including "O", "Ō", "X", "X̄" type porosity distribution patterns. The descriptions of porosity distribution patterns can be outlined as follows:

(i) "O" type distribution: In x direction, the farther away from the two edges of beam, the more porosities are. In z direction, the closer to the middle surface, the more porosities are.
(ii) "Ō" type distribution: In x direction, the farther away from the two edges of beam, the less porosities are. In z direction, the closer to the middle surface, the less porosities are.
(iii) "X" type distribution: In x direction, the farther away from the two edges of beam, the more porosities are. In z direction, the farther away from the middle surface, the more porosities are.
(iv) "X̄" type distribution: In x direction, the farther away from the two edges of beam, the less porosities are. In z direction, the farther away from the middle surface, the less porosities are.
(v) Uniform distribution: In x and z directions, the porosities are distributed uniformly.

It is assumed that all the porosity distribution patterns have the same total volume fraction of porosity α, and then the volume fraction of porosity V_p for different locations can de defined by:

$$V_p(x,z) = \lambda(x,z)\,\alpha, \tag{1}$$

where $\lambda(x,z)$ is feature function for different porosity distributions, and it can be given by

$$\lambda(x,z) = \begin{cases} \eta_1^2 \sin(\pi x/L)\cos(\pi z/h), & \text{"O" type distribution} \\ \eta_2^2 [1-\sin(\pi x/L)][1-\cos(\pi z/h)], & \text{"Ō" type distribution} \\ \eta_1\eta_2 \sin(\pi x/L)[1-\cos(\pi z/h)], & \text{"X" type distribution} \\ \eta_1\eta_2 [1-\sin(\pi x/L)]\cos(\pi z/h), & \text{"X̄" type distribution} \\ 1, & \text{uniform distribution} \end{cases} \tag{2}$$

with

$$\eta_1 = \frac{\pi}{2}, \quad \eta_2 = \frac{\pi}{\pi-2}.$$

Since two-directionally porosities are distributed in the xz plane of beam, the porous beam has the porosity-dependent mechanical properties related to x and z. The mass density ρ of two-directionally porous beam can be given by:

$$\rho(x,z) = \left(1 - V_p\right)\rho_0, \tag{3}$$

where ρ_0 is the mass density of materials without porosity.