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Finite-time output feedback tracking control for a nonholonomic wheeled mobile robot

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ABSTRACT

The finite-time tracking control problem via output feedback for a nonholonomic wheeled mobile robot with a higher-order dynamic model is investigated in this paper. To solve the problem when the robot's velocities cannot be measured, some coordinate changes are skillfully introduced at the first step. Then based on the fast finite-time control algorithm, a fast finite-time state feedback controller is designed and then a fast finite-time observer is constructed. Finally, an observer-based dynamic output feedback controller is proposed, which can guarantee that the reference trajectory can be tracked in a finite time through a rigorous stability analysis. An example is given to verify the efficiency of the proposed method.

1. Introduction

Due to the well-known Brockett Theorem [1], the control problem for a nonholonomic system is challenging and hence has attracted many attentions. As one of the benchmark nonholonomic system, many different control strategies have been employed for a wheeled mobile robot. Since it is impossible to achieve stabilization by employing any smooth or continuous time-invariant state feedback control, some other control strategies have been proposed, such as smooth time-varying control [2], discontinuous control [3], hybrid control [4], adaptive control [5], etc. Besides the stabilization problem, another interesting problem for a wheeled mobile robot is to design the trajectory tracking controller.

The work [6] solved the local tracking control problem of a mobile robot based on the linearization model. By using backstepping design, the global tracking control for mobile robots was solved in [7]. Furthermore, the work [8] considered the tracking controller design under the input saturation constraint. Based on the theory of cascaded system, the work [9] investigated the trajectory tracking control of nonholonomic systems with exponential convergence. To improve the tracking speed, recently, the finite-time control technique [10] was employed to design finite-time tracking control algorithms for wheeled mobile robots, that is the reference trajectory can be tracked in a finite time. In addition, a fascinating advantage of finite-time control lies in its good robustness, such as in [11–17]. For example, by using the theory cascaded system, the work [18] solved the global finite-time tracking control problem of a nonholonomic wheeled mobile robot with kinematics model. For the wheeled mobile robot with dynamic model, based on the technique of adding a power integrator, the work [19] proposed a higher-order finite-time tracking controller. Recently, for the formation control problem for multiple nonholonomic wheeled mobile robots, the work [20,21] studied the finite-time formation tracking control problem.

Due to the technology limitations or environment disturbances, or cost consideration, the velocity information for mobile robot is often unavailable. In this case, the observer and output feedback are required [22,23]. Compared to the state feedback control, the output feedback control for nonlinear systems is more challenging. In [24], an output feedback controller via delayed measurements was designed for a unicycle-type mobile robot. As for the finite-time output feedback control, it is more difficult even for the linear systems or lower-order nonlinear systems. The work [25] solved the finite-time output feedback stabilization for double-integrator systems. Based on the finite-time convergent observer, the output feedback controller was designed for an aircraft in [26,27].

Although there have been some results about the finite-time output control [28–30], no available result/method to the considered system in this paper, i.e., a nonholonomic wheeled mobile robot with a higher-order dynamic model. The main difficulty lies in the inherent nonlinear features. The main work/motivation of this paper is to provide a solution for the finite-time output feedback tracking control for a nonholonomic wheeled mobile robot. To solve the previous mentioned problem, first, a finite-time tracking controller via state feedback is designed based on certain coordi-

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nate changes. Then, when only the position information is available, two finite-time convergent observers are proposed to estimate the velocities in a finite time. Finally, the finite-time trajectory tracking control problem for a nonholonomic wheeled mobile robot via output feedback is solved.

2. Preliminaries and problem formulation

2.1. Problem formulation

As that in [7,19], consider the tracking control problem for a nonholonomic wheeled mobile robot with two-degrees-of freedom. The dynamic of non-holonomic mobile robot is described by:

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= \omega \\ \dot{v} &= u_1 \\ \dot{\omega} &= u_2, \end{aligned} \tag{1}$$

where (x, y) is the Cartesian position of the robot center, θ is the orientation, v and ω are the linear velocity and the angular velocity, u_1 and u_2 are the controllers.

The reference signal for the mobile robot is generated by

$$\dot{x}_d = v_d\cos(\theta_d), \qquad \dot{y}_d = v_d\sin(\theta_d), \qquad \dot{\theta}_d = \omega_d. \tag{2}$$

The control objective of this paper is to design controllers for (u_1, u_2) such that the reference trajectory $(x_d, y_d, \theta_d)^T$ can be followed by the robot's trajectory *in a finite time* under the constraint condition that the velocity information $(v, \omega)^T$ is unavailable.

2.2. Some useful definitions and lemmas

Definition 1. Denote $\mathrm{sig}^{\alpha}(x) = \mathrm{sign}(x)|x|^{\alpha}$, where $\alpha \geq 0$, $x \in R$, $\mathrm{sign}(\cdot)$ is the standard sign function. If $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is a vector, then $\mathrm{sig}^{\alpha}(x) = [\mathrm{sig}^{\alpha}(x_1), \mathrm{sig}^{\alpha}(x_2), \cdots, \mathrm{sig}^{\alpha}(x_n)]^T$.

Definition 2. Define a class of new nonlinear functions:

$$\mathrm{fsig}^{\alpha}(x) = \begin{cases} x, & \text{for } |x| > 1; \\ \mathrm{sig}^{\alpha}(x), & \text{for } |x| \leq 1, \end{cases} \tag{3}$$

where $0 < \alpha < 1$, $x \in R$. If $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is a vector, then $\mathrm{fsig}^{\alpha}(x) = [\mathrm{fsig}^{\alpha}(x_1), \mathrm{fsig}^{\alpha}(x_2), \cdots, \mathrm{fsig}^{\alpha}(x_n)]^T$.

Lemma 1 *([31]). The second-order system*

$$\ddot{x} = u, \tag{4}$$

can be globally stabilized in a finite time under the feedback control law

$$u = -k_1\mathrm{sig}^{\alpha_1}(x) - k_2\mathrm{sig}^{\alpha_2}(\dot{x}), \tag{5}$$

with $k_1, k_2 > 0, \alpha_1 \in (0, 1), \alpha_2 = 2\alpha_1/(1+\alpha_1)$.

Lemma 2 *([25]). The following second-order system*

$$\dot{e}_1 = e_2 - l_1\mathrm{sig}^{\beta_1}(e_1), \qquad \dot{e}_2 = -l_2\mathrm{sig}^{\beta_2}(e_1) \tag{6}$$

is globally finite-time stable with $l_1 > 0, l_2 > 0$, $1/2 < \beta_1 < 1$, $\beta_2 = 2\beta_1 - 1$.

3. Main results

In this section, it will be shown that the finite-time tracking control problem for mobile robot systems (1)–(2) via output feedback control is solvable. The controller design is divided into three steps. Firstly, a finite-time tracking controller via state feedback is designed. Secondly, when the robot's velocity information is unavailable, a finite-time convergent observer is constructed to estimate the unknown velocity in a finite time. Finally, an observer-based finite-time tracking controller via output feedback is given.

3.1. Design of finite-time tracking controllers via state feedback

Theorem 1. *For the mobile robot systems (1)–(2), if the controllers (u_1, u_2) are designed as*

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -l\sin\theta \\ \sin\theta & l\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} u_x + v\omega\sin\theta + l\omega^2\cos\theta \\ u_y - v\omega\cos\theta + l\omega^2\sin\theta \end{bmatrix}, \tag{7}$$

$$\begin{aligned} \begin{bmatrix} u_x \\ u_y \end{bmatrix} &= -k_1 \begin{bmatrix} \mathrm{fsig}^{\alpha_1}(x - x_d + l\cos\theta - l\cos\theta_d) \\ \mathrm{fsig}^{\alpha_1}(y - y_d + l\sin\theta - l\sin\theta_d) \end{bmatrix} \\ &- k_2 \begin{bmatrix} \mathrm{fsig}^{\alpha_2}(v\cos\theta - v_d\cos\theta_d - l\omega\sin\theta + l\omega_d\sin\theta_d) \\ \mathrm{fsig}^{\alpha_2}(v\sin\theta - v_d\sin\theta_d + l\omega\cos\theta - l\omega_d\cos\theta_d) \end{bmatrix} \\ &+ \begin{bmatrix} \dot{v}_d\cos\theta_d - v_d\omega_d\sin\theta_d - l\omega_d^2\cos\theta_d - l\dot{\omega}_d\sin\theta_d \\ \dot{v}_d\sin\theta_d + v_d\omega_d\cos\theta_d - l\omega_d^2\sin\theta_d + l\dot{\omega}_d\cos\theta_d \end{bmatrix}, \end{aligned} \tag{8}$$

then the state (x_h, y_h) will track the desired state (x_h^d, y_h^d) in a finite time, where (x_h, y_h) is a position off the wheel axis of the mobile robot by a distance l given by:

$$\begin{bmatrix} x_h \\ y_h \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \tag{9}$$

$k_1, k_2 > 0, \alpha_1 \in (0, 1), \alpha_2 = 2\alpha_1/(1+\alpha_1)$.

Proof. Under the coordinate change (9), let

$$z = [x_h, y_h]^T, \tag{10}$$

and define

$$g = \dot{z}, \qquad f = \dot{g} = \ddot{z}. \tag{11}$$

Denote

$$g = [v_x, v_y]^T, \qquad f = [u_x, u_y]^T. \tag{12}$$

Under the above notation and the coordinate change (9), it follows from the system equation (1) that

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -l\sin\theta \\ \sin\theta & l\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} u_x + v\omega\sin\theta + l\omega^2\cos\theta \\ u_y - v\omega\cos\theta + l\omega^2\sin\theta \end{bmatrix}. \tag{13}$$

Similarly, for the desired trajectory, define

$$z_d = [x_h^d, y_h^d]^T = \begin{bmatrix} x_h^d \\ y_h^d \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \end{bmatrix} + l \begin{bmatrix} \cos\theta_d \\ \sin\theta_d \end{bmatrix} \tag{14}$$

be the desired state with $g_d = \dot{z}_d$, $f_d = \dot{g}_d$.

Define

$$\bar{z} = z - z_d, \qquad \bar{g} = g - g_d, \tag{15}$$

as the tracking error, which leads to the error dynamics is

$$\begin{aligned} \dot{\bar{z}} &= \bar{g}, \\ \dot{\bar{g}} &= f - f_d. \end{aligned} \tag{16}$$