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Steepest descent quaternion attitude estimator

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ABSTRACT

A new computationally inexpensive attitude determination algorithm based on the minimization of Wahba's loss function is presented in the paper. The estimation problem is converted into quaternion representation and solved with iterative prediction–correction scheme. Unlike Kalman filter approach, an iterative gradient optimization is used to estimate the attitude quaternion and gyroscope bias. Algorithm derivation is shown and its performance is tested. The presented case study assumes configuration with three types of sensors: Sun sensors with full angular coverage, a magnetometer and a MEMS rate gyroscope. Sensor model parameters are selected to mimic a pico or nano class satellite. Orbital environment is simulated with the Bouvier–Lyddane orbit model, the IGRF magnetic field model and geometric properties of the Earth–Sun system. Periodical loss of Sun sensor data due to eclipses is taken into account. Based on the presented case study a proposition of tuning procedure and a brief comment on algorithm stability are given. The tuning approach trades off estimate convergence versus noise rejection property. In a Monte Carlo test the proposed algorithm compares well against an EKF with an attitude error within 0.1 deg in sunlight and 0.4 deg in the eclipse. Finally, a simulation showing a possibility of operating the SDQAE algorithm while sampling each of the sensors at different rate is presented.

1. Introduction

Spacecraft attitude estimation is one of the most challenging subjects in satellite guidance and navigation. Over the years many algorithms have been proposed. Approaches vary depending on the chosen attitude representation [1]. As proven by Euler in his rotation theorem, arbitrary rotation may be described by only three parameters. However, as pointed out by Stuelpnagel [2] in order to achieve globally continuous and non-singular representation of the rotation it is necessary to employ at least four components. There are some attitude determination algorithms based on singular, non redundant rotation representations. Minimal representation Extended Kalman Filters (EKF) based on Euler angles [3], Rodrigues parameters [4], and modified Rodrigues parameters [5] have been proposed. Since Wahba formulated her least squares attitude determination problem, many algorithms, such as TRIAD [6], unconstrained least-squares [7], [8], Fast Optimal Attitude Matrix (FOAM) [9], Singular Value Decomposition (SVD) [10], and EKFs [11], [12] have been created to directly estimate the attitude matrix. Since attitude matrix has as many as six redundant components, quaternions became a popular way of representing attitude. Following Davenport's Q-method solution to Wahba's problem which translates it to quaternion representation many quaternion algorithms have been proposed. Generally, they can be divided into two groups: those that solve the least-squares problem, and those based on minimum variance approach (Kalman filtering). The first group includes single-rate algorithms, for example: QUEST [13] or ESOQ [14] and other relying on recursive strategy, like Extended QUEST [15]. The second group includes, but is not limited to: Multiplicative EKF [16] and Additive EKF [17]. Additionally, there are some batch algorithms that rely on storing a certain number of past measured samples to improve the present estimate. An example of such an approach is BSEKF [18].

The following paper describes the Steepest Descent Quaternion Attitude Estimator (SDQAE) intended for satellite attitude determination. This recursive observer is based on a proposed reformulation of Wahba's problem in the quaternion space. The suggested solution is in the realm of optimization theory. The presented case study assumes availability of only three simple sensors (Sun sensor, magnetometer and rate gyroscope). This configuration is commonly used on very small spacecraft where reduction of size, mass and power requirements of the measurement suite is essential. Interestingly, presented solution turns out to be both computationally inexpensive and resilient against the periodical loses of sensor readings. It also allows performing each of the measurements at different rate, so that the more energy-consuming sensors can be operated less frequently. Inertia tensor estimate and attitude dy-

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namics model are not required, which may simplify design and implementation of the attitude determination subsystem.

The paper is organized as follows. Section 2 recalls underlying quaternion algebra concepts and introduces assumed conventions. In Section 3 the problem of attitude estimation is defined and an approach for solving it is presented. A case study is presented in Section 4 while Section 5 explains the estimator parameters tuning procedure. Results of MATLAB/Simulink simulations are discussed in Section 6. Some remarks on observer convergence are presented in Section 7, and discussion of results and plans for future work are given in Section 8.

2. Basic concepts

A unit quaternion ${}^p_r\overline{q}$ describing orientation of a frame ${}^r\mathcal{F}$ relative to a frame ${}^p\mathcal{F}$ is defined as follows:

$$ {}^p_r\overline{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^\top = \begin{bmatrix} \cos\frac{\phi}{2} \\ -u_1 \sin\frac{\phi}{2} \\ -u_2 \sin\frac{\phi}{2} \\ -u_3 \sin\frac{\phi}{2} \end{bmatrix}, \quad (1) $$

where $\mathbf{u} = [u_1\ u_2\ u_3]^\top$ is a unitary vector in the reference frame ${}^p\mathcal{F}$ describing the rotation axis and ϕ is an angle of rotation around that axis. Quaternion ${}^r_p\overline{q}$ representing an opposite relationship is equal to conjugate of ${}^p_r\overline{q}$, namely ${}^p_r\overline{q}^* = {}^r_p\overline{q} = [q_1\ {-q_2}\ {-q_3}\ {-q_4}]^\top$. A Hamiltonian quaternion product $\otimes$ can be used to express a transformation superposition, for example ${}^p_r\overline{q} = {}^s_r\overline{q} \otimes {}^p_s\overline{q}$.

It is possible to express a vector ${}^r\mathbf{v}$ defined in reference frame ${}^r\mathcal{F}$ in the other reference frame ${}^p\mathcal{F}$ using quaternion product. This calculation requires converting 3×1 vector ${}^r\mathbf{v}$ to 4×1 quaternion ${}^r\overline{\mathbf{v}}$ by introducing 0 as the first element. Then the frame change can be calculated according to equation ${}^p\overline{\mathbf{v}} = {}^p_r\overline{q}^* \otimes {}^r\overline{\mathbf{v}} \otimes {}^p_r\overline{q}$. While applying orientation quaternion algebra it is often necessary to perform normalization operation. Let us then define normalization operator $\nparallel \cdot \nparallel$ to simplify the notation of $\nparallel \overline{q} \nparallel := \overline{q}/\|\overline{q}\|$.

3. Attitude observer

The algorithm proposed in this article is inspired by the approach presented in [19], although lacks most of the simplifying assumptions that can be made if the object does not move around the Earth with a significant velocity.

One of the classical formulations of spacecraft attitude estimation is known as Wahba's problem [20]. It is defined as finding the orthonormal matrix ${}^b_iA$ which minimizes the following objective function:

$$ L({}^b_iA) = \frac{1}{2}\sum_{j=1}^{n} a_j \left\| {}^b\mathbf{d}_j - {}^b_iA\,{}^i\mathbf{d}_j \right\|^2, \quad (2) $$

where the orientation of the spacecraft body frame ${}^b\mathcal{F}$ in relation to Earth Centered Inertial (ECI) frame ${}^i\mathcal{F}$ is expressed with Direction Cosine Matrix (DCM) ${}^b_iA$. A set of n direction measurements is considered, each of them is represented by 3×1 normalized vector in the Cartesian space. Those measurements are denoted as ${}^b\mathbf{d}_j$ and expressed in the spacecraft body frame. They correspond to usually time-dependent reference directions ${}^i\mathbf{d}_j$ taken from physical models or almanacs.

Let us introduce an equivalent problem, with orientation expressed in terms of quaternion ${}^b_i\overline{q}$ instead of a matrix.

$$ L({}^b_i\overline{q}) = \frac{1}{2}\sum_{j=1}^{n} a_j \left\| f_j \right\|^2 = \frac{1}{2}\sum_{j=1}^{n} a_j \left(f_j^\top f_j \right), \quad (3) $$

where

$$ f_j = {}^b\overline{\mathbf{d}}_j - {}^b_i\overline{q}^* \otimes {}^i\overline{\mathbf{d}}_j \otimes {}^b_i\overline{q}. \quad (4) $$

Problem (3) can be solved numerically with many known optimization methods. Perhaps the simplest one is the steepest descent algorithm, where kth estimate ${}^b_i\hat{q}_k$ of orientation quaternion ${}^b_i\overline{q}_k$ is recursively improved based on its previous value ${}^b_i\hat{q}_{k-1}$ according to the equation:

$$ {}^b_i\hat{q}_k = \nparallel {}^b_i\hat{q}_{k-1} - K T_S \nabla L({}^b_i\hat{q}_{k-1}) \nparallel. \quad (5) $$

It is worth noting that subscript k refers to iteration number as well as discrete-time instant index ($t_k = kT_S$). Thus, only one iteration of optimization algorithm with step size dependent on a gain K is performed for each time period. This approach provides measurement noise filtering properties and considerably reduces computational burden. The gradient in (5) can be expressed as

$$ \nabla L\left({}^b_i\hat{q}\right) = \frac{\partial L({}^b_i\hat{q})}{\partial {}^b_i\overline{q}} = \frac{\partial}{\partial {}^b_i\overline{q}} \left(\frac{1}{2}\sum_{j=1}^{n} a_j \left(f_j^\top f_j \right) \right). \quad (6) $$

Using the formula for derivative of a product, one can write

$$ \frac{\partial}{\partial \overline{q}} \left(\frac{1}{2}\sum_{j=1}^{n} a_j \left(f_j^\top f_j \right) \right) = \frac{1}{2}\sum_{j=1}^{n} a_j \left(f_j^\top \frac{\partial f_j}{\partial {}^b_i\overline{q}} + \frac{\partial f_j}{\partial {}^b_i\overline{q}}^\top f_j \right). \quad (7) $$

After noticing that

$$ f_j^\top \frac{\partial f_j}{\partial {}^b_i\overline{q}} = \frac{\partial f_j}{\partial {}^b_i\overline{q}}^\top f_j, \quad (8) $$

one can simplify the equation (6) to:

$$ \nabla L({}^b_i\hat{q}) = \sum_{j=1}^{n} a_j \left(\frac{\partial f_j}{\partial {}^b_i\overline{q}}^\top f_j \right) = \sum_{j=1}^{n} a_j \left(J_j^\top f_j \right), \quad (9) $$

where J_j is a Jacobian matrix.

Value of gain K is set as a result of a trade-off between noise rejection capabilities (lower values of K) and convergence rate (higher values of K). If the satellite angular velocity equals zero and no noise is present in measurement signals, the estimate converges (with additional provisions – see Section 7). However, for a rotating satellite (5) cannot make the error decay asymptotically to zero. This goal may be achieved if a gyroscopic rate measurement is available. Equation (5) can be then supplemented with a prediction term resulting from a kinematic differential equation

$$ {}^b_i\dot{\hat{q}} = \frac{1}{2}{}^b_i\hat{q} \otimes {}^b\overline{\boldsymbol{\omega}}, \quad (10) $$

yielding the following formula obtained by the Euler discretization method

$$ {}^b_i\hat{q}_k = \nparallel {}^b_i\hat{q}_{k-1} - K T_S \nabla L({}^b_i\hat{q}_{k-1}) + \frac{1}{2} T_S ({}^b_i\hat{q}_{k-1} \otimes {}^b\overline{\boldsymbol{\omega}}_k) \nparallel. \quad (11) $$

The presence of the gyroscopic prediction term in (11) expands the range of admissible values for gain K. It eliminates the risk of convergence loss due to satellite spinning motion for small values of K.

In practice rate gyroscope measurements are biased. Integrating character of the prediction term (10) results in significant growth of the estimation error unless the bias is estimated and the estimate subtracted from the measured signal. To derive bias estimation formula, let us substitute ${}^b\overline{\boldsymbol{\omega}}_k$ in (11) with ${}^b\overline{\boldsymbol{\omega}}_k + {}^b\overline{\mathbf{e}}_k$ to