Aerospace Science and Technology 77 (2018) 180–188

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Reachable set computation for spacecraft relative motion with energy-limited low-thrust

Sangjin Lee *, Inseok Hwang

Purdue University, West Lafayette, IN, 47907, United States

ARTICLE INFO

Article history:
Received 4 April 2017
Received in revised form 21 December 2017
Accepted 20 February 2018
Available online 2 March 2018

ABSTRACT

For spacecraft in formation flight, the knowledge on a reachable set for their relative motion is crucial as it can be used to 1) assess their operational capability and plan tasks accordingly, and 2) predict the operational boundary of neighboring spacecraft, from which the collision probability can be effectively monitored, improving the level of situational awareness. In this paper, a new approach for reachable set computation is proposed that computes accurate inner and outer approximations of the reachable set for a spacecraft's relative motion with energy-limited low thrust. Finding the exact boundary of the reachable set requires solving an optimal control problem with infinitely many sets of initial and terminal conditions, which is intractable. To overcome this difficulty, an analytical solution to the optimal control problem is introduced, and an ellipsoidal approximation method is applied to the solution to find two inner and outer ellipsoids that approximate the exact boundary of the reachable set. The effectiveness of the proposed approach is demonstrated with illustrative numerical examples.

1. Introduction

Due to the rapid advancement of autonomous spacecraft, many formation flying missions have become increasingly automated, enabling low-cost, highly efficient, and flexible operation of space assets. The use of these autonomous spacecraft in formation flight has introduced fundamental research problems such as formation keeping, reconfiguration, and collision avoidance, for which a plenty of work has been performed [1–3]. However, as formation flying missions and the corresponding tasks have become increasingly complicated involving a growing number of spacecraft and thus had more stringent requirements, there has been a need for more advanced research that enables to develop smarter autonomous spacecraft, having improved autonomy and intelligence.

As discussed in the 2015 NASA Technology Roadmaps [4], one of the key characteristics that the advanced autonomous spacecraft should possess is the capability of maintaining high-level cognitive awareness about the operational capabilities and limitations of the spacecraft itself and its environment (e.g., neighboring space objects). This cognitive information is crucial especially for spacecraft in formation flight as it can be used for 1) intelligent mission planning: the spacecraft can autonomously evaluate the feasibility of assigned missions and, if necessary, re-plan the missions, and 2) effective space situational awareness: the spacecraft can accurately predict the operational envelop of its environments (e.g., region reachable by an adjacent spacecraft in a cluster having maneuver capability), from which the collision probability can be monitored with the objective of collision avoidance. From this perspective, we aim to develop a methodology that improves the level of the cognitive awareness about the relative motion of spacecraft in formation flight.

In general, a system's operational envelope can be effectively represented by reachable sets in the system's state space, which denote all possible behaviors of the system given inputs and initial conditions and thus its boundary represents the limit of the system's operational capacity [5,6]. In control literature, research on reachable set computation has been extensively performed mostly based on optimal control theory, which requires finding the exact or approximate solution of the Hamilton–Jacobi–Isaacs partial differential equation, resulting in various techniques applicable to certain types of dynamical systems (for an extensive literature review, see [7] and references therein).

A few efforts have been also made to compute the reachable set for a spacecraft either by applying the existing techniques in control literature or devising new methods tailored to specific space applications. For example, Xue et al. [8] developed a method that computes an approximated upper bound on the reachable domain for a spacecraft capable of initiating a single impulse. In [9,10], Wen et al. also considered the motion of a spacecraft with a single impulse and provided an accurate envelope of the reachable

* Corresponding author.
E-mail addresses: lee997p@gmail.com (S. Lee), ihwang@purdue.edu (I. Hwang).

https://doi.org/10.1016/j.ast.2018.02.034
1270-9638/

set in three-dimensional space. Holzinger et al. [11,12] computed an operational range for an on-orbit spacecraft using Gauss's variational equations and represented it using classical orbital elements. These approaches, however, are based on the spacecraft's absolute motion about the Earth (i.e., the two body dynamics) and thus not directly applicable to the spacecraft's relative motion. A couple of approaches have been proposed for reachable set computation for a spacecraft's relative motion. Holzinger and Scheeres extended the existing reachability and optimal control theory to a class of nonlinear systems for cases where they have ellipsoidal initial sets [13,14], and applied them to various space reachable set computation problems including nonlinear relative orbital motion. Wen and Gurfil [15] proposed a method to compute a so-called relative reachable domain for a spacecraft's relative motion given initial state uncertainties. HomChaudhuri et al. computed reach-avoid sets for spacecraft docking applications using hybrid system approach [16]. In [17,18], Lee et al. developed analytical solutions to stochastic reachable set computation that account for uncertainties in initial sets and external control inputs. These approaches, however, also have limitations in the sense that 1) they are computationally expensive [13,14], or 2) they assume no external control [15], assume piecewise constant thrusts [16], or use probabilistic models for control input [17,18], which need to be modified for reachability analysis for cases where there exists external control with deterministic bound (which is the case in many practical situations).

In this research, we aim at developing a new reachable set computation method for spacecraft relative motion that can account for explicit bounds on energy of external control applied to a spacecraft, which can overcome the limitations of the aforementioned approaches. In particular, we focus on continuous low-thrust as the external control in the sense that, in many formation flying missions, low-thrust propulsion systems have been widely used due to their high specific impulse and thus high fuel efficiency (see [1] and references therein). To achieve this goal, we first formulate the reachable set computation problem using optimal control theory, resulting in an optimal control problem with infinitely many sets of initial and terminal conditions that need to be solved to find the exact boundary of the reachable set. Since finding the solutions of the optimal control problem for all the infinitely many boundary conditions is intractable, we propose to find inner and outer approximations of the reachable set, which can accurately and efficiently approximate the boundary of the reachable set without significant computational loads. To this end, we first derive an analytical solution to the optimal control problem, from which we observe an analytical structure that the cost function of the optimal control problem can be represented as a high-dimensional ellipsoid function of the initial and terminal conditions. From this observation, we propose to apply an ellipsoidal approximation technique [19,20] that provides a systematical way to compute both inner and outer ellipsoids that approximate the solutions of the optimal control problem with the infinitely many initial and terminal conditions (i.e., the exact boundary of the reachable set).

The paper is organized as follows. In Sec. 2, the reachable set computation problem is mathematically formulated as an optimal control problem, given relative motion dynamics and control bound. In Sec. 3, inner and outer approximations of the reachable set are derived using an analytical solution to the optimal control problem as well as an ellipsoidal approximation technique. In Sec. 4, the proposed approach is demonstrated with illustrative numerical examples, and conclusions are presented in Sec. 5.

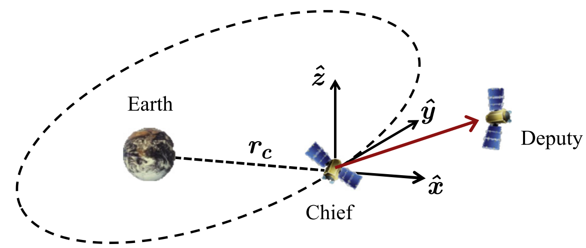

Fig. 1. Relative motion in the local-vertical-local-horizontal frame.

2. Problem formulation

2.1. Spacecraft relative motion dynamics

In this study, we consider the linearized relative motion of a deputy satellite with respect to a chief satellite that moves around the Earth in a near-circular orbit. Defining $x(t)$, $y(t)$, and $z(t)$ as the coordinates of the deputy's position at time t relative to the chief in the local-vertical/local-horizontal frame (see Fig. 1), the relative motion is described by the well-known Hill–Clohessy–Wiltshire (HCW) equations as [21]

$$\begin{aligned} &\ddot{x}(t) - 2n\dot{y}(t) - 3n^2 x(t) = u_x(t) \\ &\ddot{y}(t) + 2n\dot{x}(t) = u_y(t) \\ &\ddot{z}(t) + n^2 z(t) = u_z(t) \end{aligned} \tag{1}$$

where $u_x(t)$, $u_y(t)$, and $u_z(t)$ are the control input applied to the deputy; $n = (\mu/r_c^3)^{1/2}$ is the mean orbital motion of the chief; $r_c(t)$ is the orbital radius of the chief from the center of the Earth; μ is the gravitational constant of the Earth. By defining the deputy's state vector $\mathbf{x}(t)$ and the control vector $\mathbf{u}(t)$ as $\mathbf{x}(t) = [\mathbf{r}(t)^T \ \mathbf{v}(t)^T]^T$ and $\mathbf{u}(t) = [u_x(t) \ u_y(t) \ u_z(t)]^T$ (where $\mathbf{r}(t) = [x(t) \ y(t) \ z(t)]^T$ and $\mathbf{v}(t) = [\dot{x}(t) \ \dot{y}(t) \ \dot{z}(t)]^T$ are the position and velocity vectors, respectively), the state-space equation associated to Eq. (1) is obtained as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{2}$$

where

$$\begin{aligned} &\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{I}_{3\times3} \end{bmatrix} \\ &\mathbf{A}_1 = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}, \qquad \mathbf{A}_2 = \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

Define $\boldsymbol{\Upsilon}(t)$ and $\boldsymbol{\Gamma}(t, t_0) \equiv \boldsymbol{\Upsilon}(t)\boldsymbol{\Upsilon}(t_0)^{-1}$ as the fundamental matrix and state transition matrix associated to $\mathbf{A}$, respectively. Then, it is well known that the solution of Eq. (2), given the initial condition $\mathbf{x}(t_0)$ and control history $\mathbf{u}(t)$, is obtained by

$$\mathbf{x}(t) = \boldsymbol{\Gamma}(t, t_0)\mathbf{x}(t_0) + \int_{t_0}^{t} \boldsymbol{\Gamma}(t, \tau)\mathbf{B}\mathbf{u}(\tau)d\tau \tag{4}$$

2.2. Reachable set

Note that the goal of this study is to compute the boundary of the set of all possible states (called *the reachable set*) of a spacecraft that can be reached by a continuous low-thrust propulsion, given 1) a range of the initial state of the spacecraft, 2) an energy bound on the thrust, and 3) a constraint on maneuver time duration. In what follows, the range of the initial state and energy bound are discussed in detail, and the reachable set is formally defined.