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Equations of motion for optimal maneuvering with global aerodynamic model

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Aircraft point-mass equations of motion have been largely adopted to calculate optimal trajectories with local aerodynamic models, i.e. valid in a restricted domain. However, some optimal maneuvers may need aerodynamic models valid for a broader range of flight conditions. For this purpose, global aerodynamic models are attractive but their nonlinear structure can preclude obtaining optimal trajectories by an indirect method together with the point-mass equations of motion. To solve this impasse without resorting to direct methods the authors propose a new set of aircraft equations of motion. When compared to the point-mass equations, the proposed set permits the inclusion of the angular velocity in the evaluation of aerodynamic forces, making them more accurate. Another advantage of the proposed model over the point-mass one is that it allows a qualitative estimate of the control surface deflections after the trajectory is obtained, which enables to discard solutions with infeasible deflections. To verify consistency, the proposed equations of motion are compared by simulation to the point-mass and to the rigid-body equations. The use of the proposed set of equations is demonstrated by three optimizations of a 360° roll problem.

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1. Introduction

In the indirect approach to the optimal control problem, the performance index is indirectly optimized by the solution of the boundary value problem (BVP) resulting from optimal necessary conditions. Some of these conditions require that the modeling functions of the aircraft to be continuous. In this context, nonlinear aerodynamic models derived from lookup table data using multivariate orthogonal functions [1–4] present at least two major advantages. The first is the globality, since an optimal maneuver typically does not remain close to a single operating point. The second is the analytical differentiability, required to satisfy the optimal necessary conditions of the indirect approach.

Ideally, optimal maneuvers should be evaluated with the rigid-body equations to result in optimal control deflections. However, the use of this set of equations within the indirect method is a hard numerical task since the resulting boundary value problem has a high number of differential equations. In the literature there are a few results of trajectory optimization applying the rigid-body model and they are mostly limited to the motion contained in the vertical plane [5–7], reducing the number of differential equations by half. The complete rigid-body equations were used by Fan et al. [8] to study time-optimal lateral maneuvers but they resorted to a direct method, which finds the optimal control by directly minimizing the performance index. To the best knowledge of the authors, this is the only publication with the complete rigid-body equations.

On the other hand, point-mass equations of motion have been largely used in trajectory optimization [9–20], including aerobatic maneuvers [21–24], mainly because of their reduced number of differential equations compared to the rigid-body model. Combined with global aerodynamic models, however, some issues arise to satisfy the optimal necessary conditions and to accurately evaluate the aerodynamic forces. To fix that, the authors propose an intermediate model, suitable for the study of optimal maneuvers, that has a lower order compared to the rigid-body model while retaining better accuracy than the point-mass equations in the aerodynamics calculation.

This paper is structured as follows: Section 2 discusses the optimal control problem of interest and the corresponding BVP that results from the optimal necessary conditions. In Sec. 3 the F-16 aircraft model is described. The aerodynamic model is then analyzed in Sec. 4 with the point-mass equations of motion within the presented optimal control problem and the issues are exposed. As consequence, a new set of equations of motion is proposed and compared by simulation to the point-mass and the rigid-body. To

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Nomenclature

c.g.	center of gravity	s	order of the equality constraint
$\boldsymbol{f}$	state function	t	time .... s
m	number of control variables	$\boldsymbol{u}$	control
n	number of state variables	V	airspeed .... ft/s
o	number of final state constraints	$\boldsymbol{x}$	state
$\mathbb{R}$	set of real numbers	$\mathbb{Z}^+$	set of positive integers

exemplify the numerical application of the proposed equations of motion in maneuvering optimization, the complete roll around the longitudinal axis maneuver is optimized in Sec. 5.

2. Optimal control problem: the indirect method

Consider the optimal problem of finding the control history $\boldsymbol{u}: [t_0, t_f] \to \mathbb{R}^m$ that minimizes the following performance index:

$$J(\boldsymbol{u}) := \Phi(t_f, \boldsymbol{x}(t_f)) + \int_{t_0}^{t_f} L(t, \boldsymbol{x}, \boldsymbol{u})\, dt, \tag{1}$$

subject to a dynamic constraint on the state trajectory $\boldsymbol{x}: [t_0, t_f] \to \mathbb{R}^n$:

$$\dot{\boldsymbol{x}} = f(t, \boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{f}: \mathbb{R}^{1+n+m} \to \mathbb{R}^n; \tag{2}$$

an initial state condition at the given initial time t_0:

$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0, \quad \boldsymbol{x}_0 \in \mathbb{R}^n; \tag{3}$$

and a final state function constraint:

$$\boldsymbol{\Psi}(t_f, \boldsymbol{x}(t_f)) = \boldsymbol{0}, \qquad \boldsymbol{\Psi}: \mathbb{R}^{1+n} \to \mathbb{R}^o, \quad o \le n, \tag{4}$$

where $L(t, \boldsymbol{x}, \boldsymbol{u})$ is the weighting function, $\Phi(t_f, \boldsymbol{x}(t_f))$ is the final weighting function, $\boldsymbol{\Psi}(t_f, \boldsymbol{x}(t_f))$ is the fixed final function. Both Φ and $\boldsymbol{\Psi}$ are functions of the final state $\boldsymbol{x}(t_f)$, but the first one is to be optimized in the performance index, Eq. (1), whereas the second is to be satisfied in the final boundary condition, Eq. (4). In trajectory optimization, the state differential equations Eq. (2) are the so called equations of motion.

The indirect methods are based on the application of the calculus of variations to transform the optimal control problem formulated above in a BVP. This way, a Hamiltonian H and an auxiliary function Θ are defined as:

$$H(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) := L(t, \boldsymbol{x}, \boldsymbol{u}) + \boldsymbol{\lambda}^\top \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}), \tag{5}$$

$$\Theta(t_f, \boldsymbol{x}(t_f), \boldsymbol{\nu}) := \Phi(t_f, \boldsymbol{x}(t_f)) + \boldsymbol{\nu}^\top \boldsymbol{\Psi}(t_f, \boldsymbol{x}(t_f)), \tag{6}$$

where $\boldsymbol{\lambda}: [t_0, t_f] \to \mathbb{R}^n$ is the costate and $\boldsymbol{\nu} \in \mathbb{R}^o$ are Lagrange multipliers. Then, the resulting necessary conditions are [25]: the stationarity condition

$$\frac{\partial H(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda})}{\partial \boldsymbol{u}} = 0; \tag{7}$$

the costate differential equation

$$\dot{\boldsymbol{\lambda}}^\top = -\frac{\partial H(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda})}{\partial \boldsymbol{x}}; \tag{8}$$

the final costate condition

$$\boldsymbol{\lambda}^\top(t_f) = \frac{\partial \Theta(t_f, \boldsymbol{x}(t_f), \boldsymbol{\nu})}{\partial \boldsymbol{x}(t_f)}; \tag{9}$$

and an additional final condition if the final time t_f is free

$$\frac{\partial \Theta(t_f, \boldsymbol{x}(t_f), \boldsymbol{\nu})}{\partial t_f} + H(t_f, \boldsymbol{x}(t_f), \boldsymbol{u}(t_f), \boldsymbol{\lambda}(t_f)) = 0. \tag{10}$$

The stationarity condition, Eq. (7), applies while the control does not reach its boundaries. It is a particular case of the Pontryagin Maximum Principle which states that the Hamiltonian must be minimized over the set U of all feasible control, i.e. $\boldsymbol{u} \in U \subset \mathbb{R}^m$ [25].

Summarizing, the indirect method approach to the optimal control problem must solve a BVP with: state differential equations, Eq. (2); state initial condition, Eq. (3); costate differential equations, Eq. (8); and final conditions, Eqs. (4), (9), and (10). Therefore, an intrinsic difficulty of the indirect method is that the resultant BVP has twice differential equations than the dynamic system being optimized. Another drawback is that one has to provide an initial guess to the algorithm that will solve numerically the BVP and this can be a difficult task, specially for the costate. Many researchers have tried to overcome this difficulty estimating the costate from direct methods, among which we mention the works by Stryk and Burlisch [26], Seywald and Kumar [27], Grimm and Markl [28], and Fahroo and Ross [29]. Rather than depending on direct methods results, Graichen and Petit [30] developed a methodology that begins in a simpler auxiliary problem where the costate is null and recursively transform it into the original problem.

It is worth noting that the optimal control problem stated in this section and applied in this paper produces only optimal open-loop trajectories. To implement these trajectories in practice it is necessary to use them posteriorly as references to a closed-loop control.

2.1. Optimal control with equality constraint on a function of the state

Many times constraints must be included in the optimal control problem. They can be either equality or inequality, function of the control, or function of the state, or even function of the control and the state, among others types of restrictions. In this paper, only the equality constraint on a function of the state will be addressed, which is given by:

$$\Gamma(t, \boldsymbol{x}) = 0, \quad \Gamma: \mathbb{R}^{1+n} \to \mathbb{R} \tag{11}$$

The following development is given by Bryson and Ho [25] and is reprinted here only for completeness. As the function Γ is not explicit on control $\boldsymbol{u}$, successive time derivatives of Γ are held until an explicit dependent expression is obtained. If s derivatives are needed, Eq. (11) is an equality constraint of sth-order. For Eq. (11) to remain valid along time, it's derivatives must also be null, therefore:

$$\Gamma^{(s)}(t, \boldsymbol{x}, \boldsymbol{u}) = 0 \tag{12}$$

Appending $\Gamma^{(s)}(t, \boldsymbol{x}, \boldsymbol{u})$ (the sth time derivative of S) to the Hamiltonian:

$$H(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}, \mu) = L(t, \boldsymbol{x}, \boldsymbol{u}) + \boldsymbol{\lambda}^T \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) + \eta \Gamma^{(s)}(t, \boldsymbol{x}, \boldsymbol{u}) \tag{13}$$

where $\eta: [t_0, t_f] \to \mathbb{R}$ is the Lagrange multiplier of the constraint.

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