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# Two-degree-of-freedom attitude tracking control for bank-to-turn aerial vehicles: An additive-state-decomposition-based method

Jinrui Ren [a,*], Quan Quan [a], Li-Bing Zhao [b], Xunhua Dai [a], Kai-Yuan Cai [a]

[a] *School of Automation Science and Electrical Engineering, Beihang University, Beijing, 100191, China*
[b] *China Academy of Aerospace Standardization and Product Assurance, Beijing, 100071, China*

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A B S T R A C T

This paper presents an additive-state-decomposition-based attitude tracking control method for a class of bank-to-turn aerial vehicles subject to unknown disturbances and nonlinear coupling. This method 'additively' decomposes the original tracking problem into two more tractable problems, namely a tracking problem for a deterministic nonlinear 'primary' system, and a disturbance rejection problem for a linear time-invariant 'secondary' system. Based on the decomposition, a backstepping controller is designed for the primary system to track the reference attitude signal, and a proportional-integral controller is applied to the secondary system to compensate for the disturbances. Finally, the two designed controllers are combined to achieve the original control objective. By using additive state decomposition, the proposed control method with two degrees of freedom can consider tracking task and disturbance rejection task respectively. Simulation results illustrate that the proposed controller can track the reference attitude signal and compensate for disturbances meanwhile. Additionally, the ASD-based controller outperforms the traditional backstepping controller in the presence of unknown disturbances and input delay, and the robustness of the full system can be improved by adjusting the controller parameters of the secondary system.

## 1. Introduction

It is well-known that the bank-to-turn (BTT) control provides potential performance improvement for aerial vehicles including missiles [1] and unmanned aerial vehicles (UAVs) [2]. Compared with the skid-to-turn (STT) control mode, the BTT control mode provides higher maneuverability, larger acceleration, and faster response. Hence, the autopilot design of BTT aerial vehicles (BTT aerial vehicles denote the aerial vehicles adopting the BTT control mode) has received widespread attention. To perform the control strategies of BTT autopilots, aerial vehicles must have the capability of changing the orientation of acceleration rapidly via a considerably large roll rate. However, such a large roll rate further induces unignorable cross-coupling, which results in undesirable pitch and yaw motions. Furthermore, the imprecise knowledge of aerodynamic parameters, highly nonlinear dynamics, and unknown disturbances make the autopilot design of BTT aerial vehicles more challenging [3,4]. Thus, the autopilot design of BTT aerial vehicles is meaningful.

Various control methods have been applied to the autopilot design of BTT aerial vehicles. The most frequently-used methods can be divided into five categories.

(i) The gain-scheduling control method is adopted by [5–7]. It is a linear control method, which is based on some classical linear control methods, such as linear quadratic regulator, $H_\infty$, $\mu$-synthesis. Two significant limitations of gain-scheduling are that a linearization assumption is needed and parameter variations may be too fast. In order to solve the first limitation, a linear parameter varying (LPV) model is adopted by [7]. However, the control design is separated onto decoupled channels to facilitate the transformation into an LPV form. Since BTT aerial vehicles have high coupling characteristic, the methods aiming at linear independent channel design cannot respond very well due to a large roll rate.

(ii) The input/output feedback linearization control method is one of common nonlinear methods [8,9], which is proposed for multiple-input–multiple-output systems directly. A kind of feedback linearization technique along with a singular perturbation-like technique is adopted by [9], and excellent set-point tracking performance is obtained. The drawbacks of feedback linearization are that an accurate model is often required and the intrinsic singularity problem may occur.

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* Corresponding author.
*E-mail address:* renjinrui@buaa.edu.cn (J. Ren).

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(iii) The backstepping control method [2,10] is another choice, which is a systematic approach guaranteeing stability based on Lyapunov functions. However, system performance may deteriorate when model uncertainty is large. In order to solve the problem, robust sigmoid-like control functions are used to confine the uncertain terms [10]. However, in practice, uncertain terms are mostly related to aerodynamic coefficients and dynamic pressure. Therefore, the corresponding bounding functions, which are often constructed using experience or based on a priori knowledge on system behaviors, are difficult to obtain.

(iv) In order to cope with various disturbances, the robust control method including $H_\infty$ [11], $\mu$-synthesis [5], and sliding mode control [12,13] attract researchers' attention. Robust control allows the formulation of the robustness and performance requirements with respect to plant uncertainty or disturbances. The resulting controllers can cover a wider flight envelope and offer a systematic treatment of coupled system dynamics. Nevertheless, robust control is less intuitive than classical control techniques. In addition, a disturbance observer is another choice to improve system robustness [14].

(v) The intelligent control method is also popular, such as fuzzy logic [4,15] or neural networks [3,16] based control methods. Intelligent control can tackle unknown nonlinearities, but the corresponding design is somewhat complicated. The number of fuzzy rules may become prohibitively large and sparse, and neural networks may cost a long time to learn for complex high-dimensional systems.

It is well-known that there is an intrinsic conflict between performance (trajectory tracking) and robustness (disturbance rejection) in the standard feedback framework [17]. This conflict inspires an idea that it would be better to tackle the two control objectives separately. In order to exploit this idea, an additive-state-decomposition-based (ASD-based) [18] attitude tracking control method is proposed for the extended medium range air-to-air technology (EMRAAT) BTT missile, a typical representative of BTT aerial vehicles. The basic idea of the control design is to additively decompose the original tracking problem into two more tractable problems, namely a tracking problem for a deterministic nonlinear primary system and a disturbance rejection problem for a linear time-invariant (LTI) secondary system. Based on the decomposition, a backstepping controller is designed for the tracking problem, and a classical proportional-integral (PI) controller is adopted to solve the disturbance rejection problem for the secondary system. The advantage of the ASD-based control method lies in the decomposition of the original problem into two well-solved control problems. Unlike the aforementioned control methods, the proposed control method avoids model linearization and neglect of nonlinear dynamics as much as possible.

The main contributions of this paper are summarized as below.

(i) An ASD-based control method is proposed to solve the attitude tracking problem for BTT aerial vehicles. Introducing additive state decomposition simplifies the design and also increases the flexibility of controller design.

(ii) The proposed control method is a type of two-degree-of-freedom control method, which separates a disturbance rejection task from a tracking task. Thus, it becomes convenient to consider tracking performance and robustness, respectively, and it is easier to obtain a better comprehensive performance.

(iii) A major difference of this control method from the previous ASD-based control method [18,19] is the tracking task is allocated to a nonlinear system, rather than an LTI system. On the other hand, allocating disturbances to an LTI system makes disturbance rejection more achievable. Moreover, only state feedback controllers are designed for nonlinear subsystems in [18,19], whereas a backstepping controller is adopted for the nonlinear subsystem in this paper.

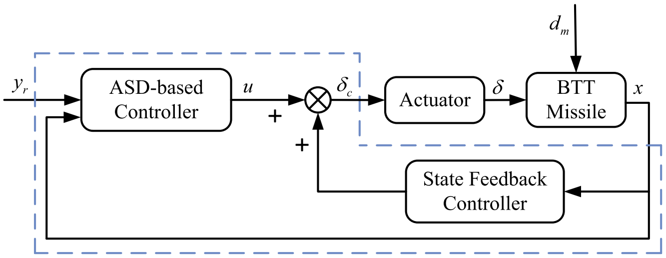

**Fig. 1.** Schematic diagram of the BTT missile system.

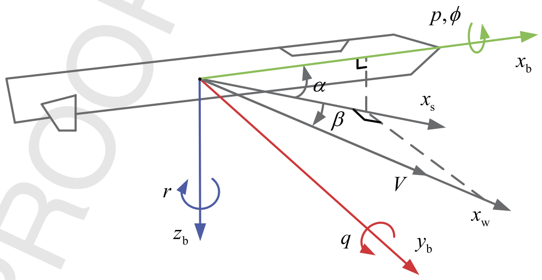

**Fig. 2.** BTT missile diagram.

The remainder of this paper is organized as follows. In Section 2, an EMRAAT BTT missile model is given. Section 3 presents the ASD-based tracking controller design for the BTT missile. In Section 4, simulations are carried out to demonstrate the effectiveness and robustness of the proposed control method. Section 5 concludes this paper and gives future work.

## 2. EMRAAT BTT missile model

A schematic diagram of the complete BTT missile system is shown in Fig. 1, where $d_m$ is the disturbance acting on the BTT missile. The BTT missile model will be described in this section, and the BTT autopilot including the state feedback controller and the ASD-based controller will be designed in the subsequent section.

The studied BTT aerial vehicle is an EMRAAT BTT missile, a typical benchmark. When establishing the BTT missile model, three commonly used coordinate frames are the missile-body frame ($o_b x_b y_b z_b$), the wind frame ($o_w x_w y_w z_w$), and the stability frame ($o_s x_s y_s z_s$), which are shown in Fig. 2. Several variables necessary for the later model representation are also displayed in Fig. 2, where $\alpha$ is the angle of attack, $\beta$ is the sideslip angle, $\phi$ is the roll angle, $p$ is the roll rate, $q$ is the pitch rate, and $r$ is the yaw rate. The concrete definitions of the mentioned coordinate frames and variables can be found in [20].

The dynamic equations of the EMRAAT BTT missile in a flight condition of Mach 2 and 30,000 ft are given as below

$$\begin{aligned}
\dot{\alpha} &= q - \tan(\beta)\left[p\cos(\alpha) + r\sin(\alpha)\right] + \frac{0.0166}{\cos(\beta)}\cos(\alpha)\cos(\phi) \\
&\quad - \frac{\cos(\alpha)}{\cos(\beta)}\left(0.092\alpha + 3.654\times10^{-5}q + 0.01516\delta_q\right) \\
\dot{\beta} &= p\sin(\alpha) + (-0.0375\beta - 1.8396\times10^{-6}p \\
&\quad + 0.000504\delta_p - 0.00882\delta_r)\cos(\beta) - r\cos(\alpha) \\
&\quad + 0.0166\sin(\phi)\cos(\beta) \\
\dot{\phi} &= p \\
\dot{p} &= 1.7919\times10^{-5}p^2 + 0.0184q^2 - 0.0184r^2 \\
&\quad - 0.0151pq - 0.0023pr
\end{aligned}$$

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