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Optimal formation reconfiguration of satellites under attitude constraints using only thrusters

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ABSTRACT

An optimal formation reconfiguration method under the constraints of a satellite attitude with respect to an inertial frame is addressed. Both the satellite position and attitude are controlled by only two body-fixed thrusters for an in-plane maneuver. To tackle the underactuated control problem, an attitude controller for tracking reference accelerations is firstly derived on the basis of Lyapunov approach. This controller allows us to consider the attitude constraints as input directional constraints because the satellite attitude is controlled so that the thrust direction is coincide with the force direction required for the orbit transfer. Secondly, a formation reconfiguration method based on the Fourier series is used as the reference inputs, and boundary conditions that make the resulting input trajectory an ellipse are shown. Such elliptic input trajectory changes the input direction monotonically, which enables bounding it around an desired direction. The proposed underactuated-controller achieves a reconfiguration maneuver while keeping the satellite attitude within a range from a specified direction, and thus is useful when several thrusters of a satellite fail due to malfunctions. Finally, numerical simulation results validate the effectiveness of the proposed relocation method by comparing energy consumptions and bounded satellite attitude angles.

1. Introduction

Formation flying is one of promising technologies for space missions [1–4], in which several satellites are orbiting on a close formation and controlled to keep their relative position and attitude to one another. The relative motion of a satellite, called “follower”, with respect to a “leader” satellite have been discussed using linearized equations. The equations of motion of the follower in the proximity of the leader is expressed with Hill–Clohessy–Wiltshire (HCW) equations [5] for a circular orbit and Tschauner–Hempel equations [6] for an elliptic orbit. These linearized equations have periodic solutions and they are useful to design satellite formation relocation or rendezvous trajectories [7–9]. From a practical viewpoint, optimal trajectories to desired relative orbits should be designed to minimize energy or fuel consumption. In terms of energy optimality, Carter and Pardis [10] propose an optimal feedback controller for a rendezvous problem in a circular orbit under constraints of bounded thrusts. Palmer [11] analytically shows an optimal controller based on the Fourier series for relocating a follower satellite to a desired relative orbit. This method is extended to study analytical solutions for optimal formation reconfigurations under J_2 perturbation [12] as well as to derive an optimal controller for formation flying in an elliptic orbit [13]. For both energy and fuel optimizations, Xi and Li [14] show an optimal reconfiguration method in an elliptic orbit using a homotopy method. Though these methods enable optimal formation control, arbitrary magnitudes of accelerations are assumed available in any directions. This assumption implies that enough number of thrusters are equipped on satellites, and thus the prior controllers are not applicable when some thrusters have failed or a satellite equips a few number of thrusters.

Even if a follower satellite equips enough number of thrusters, the satellite attitude control throughout a reconfiguration maneuver is needed for practical mission requirements. Moreover, the satellite attitude depends on the thrust directions when only a few number of thrusters are available. In that case, the thrust directions must be oriented along desired directions by controlling the satellite attitude. This indicates that the requirements on the attitude angles can be equivalently considered as input directional constraints. Mitani and Yamakawa [15] show an optimal rendezvous method under thrust directional constraints with respect to a leader satellite. The optimal controller is based on a satisficing method [16] for keeping the thrust direction within an allowable area. The control method in [15] is further extended to an optimal controller [17] in terms of L_1 and L_2 optimizations of

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thruster accelerations by the use of a smoothing method. Guelman et al. [18] deal with a formation control under a single input constraint in a circular orbit. Recent research deals with position and attitude control of satellites in formation flying [19–21]. These studies, however, assume that the satellite attitude can be controlled arbitrarily. That is, the attitude dynamics of a follower satellite is not explicitly studied. Therefore the existing control methods may affect the attitude motion in the formation flying and are not applicable to the system in this paper.

This study aims to show an optimal formation reconfiguration method under a satellite attitude constraint with respect to an inertial frame. In the reconfiguration maneuver for in-plane motion, both the satellite position and attitude are controlled using only two thrusters. For such an underactuated satellite, its attitude angle must be controlled for generating thruster forces in desired directions. Thus this study firstly derives an attitude tracking controller using the thruster inputs on the basis of Lyapunov stability. The derived tracking controller reduces the reconfiguration problem under the attitude constraint to the one under a thrust directional constraint. Secondly, reference inputs for the tracking method is designed. Then, the conditions of the reference inputs to bound the thrust direction around desired one are discussed. The maximum bound of the attitude angle can be accurately estimated. Numerical simulation results verify the effectiveness of the proposed controller and the accuracy of the estimated bounds, and compare the energy consumptions. The proposed underactuated control method uses two thrusters for the reconfiguration maneuver while keeping the satellite attitude in a certain region. The proposed method is useful and applicable for formation flying maneuvers under attitude constraints such as electric power generation with fixed solar array panels, coronagraph observations [3,22] or communication with ground stations. The formation reconfiguration with only two thrusters also indicates that the number of actuators required for formation flying can be reduced and is useful when several thrusters of a satellite fail due to malfunctions.

This paper is organized as follows. Section 2 denotes the relative equations of a follower satellite in a near-circular orbit and their analytical solutions. Modal equations are also shown to simplify the interpretation of the reconfiguration problem. Section 3 describes an optimal reconfiguration method and its boundary conditions to orient the satellite attitude along a desired direction. Furthermore, an initial input direction and the estimation of the attitude bound are analyzed. Finally, some numerical simulation results are shown to verify the effectiveness of the proposed method in Section 4, and Section 5 concludes this paper.

2. Problem formulation

2.1. Hill–Clohessy–Wiltshire equations

The relative motion of a follower satellite is described in a leader-fixed frame. In the leader-fixed coordinate system, x-axis lies in the radial direction from the Earth, z-axis points to the orbital momentum vector of the leader, and y-axis completes the right-handed frame. Since the cross-track motion along the z-axis is decoupled from the in-plane motion, this study considers a formation reconfiguration in the x–y plane. If cross-track motion exists, the translational and rotational motion along the z-axis should be firstly controlled so that the proposed method in this paper can be applied. The in-plane equations of motion are written as

$$\ddot{x} = 2\Omega\dot{y} + \Omega^2 (R_l + x) - \frac{\mu (R_l + x)}{\left((R_l + x)^2 + y^2 + z^2\right)^{\frac{3}{2}}} + u_x \tag{1}$$

$$\ddot{y} = -2\Omega\dot{x} + \Omega^2 y - \frac{\mu y}{\left((R_l + x)^2 + y^2 + z^2\right)^{\frac{3}{2}}} + u_y \tag{2}$$

where Ω, R_l, and μ are the orbital rate of the leader satellite, the orbital radius of the leader, and the gravitational constant, respectively. The variables (x, y, z) and (u_x, u_y) denote the relative position of the follower and external accelerations. Although practical disturbances such as atmospheric drag and/or solar radiation pressure should be considered in the external accelerations, this study ignores these effects to simplify the simultaneous control of the relative position and attitude. The extension of the proposed method to the motion under the disturbances will be studied in future works.

Assuming that the orbital radius of the leader is much larger than the distance between the leader and follower, we obtain linearized equations, i.e., HCW equations [5] as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \Omega x \\ \dot{x} \\ \Omega y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & \Omega & 0 & 0 \\ 3\Omega & 0 & 0 & 2\Omega \\ 0 & 0 & 0 & \Omega \\ 0 & -2\Omega & 0 & 0 \end{bmatrix}\begin{bmatrix} \Omega x \\ \dot{x} \\ \Omega y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{3}$$

$$\Rightarrow \dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}_{xy} \tag{4}$$

Note that the variables x and y in the state vector $\boldsymbol{x}$ are multiplied by the orbital rate Ω to simplify analytical solutions shown in the followings.

The analytic solutions of the HCW equations with no external forces, i.e., homogeneous solutions, are described as follows.

$$\boldsymbol{x}_h(t) = \Phi(t)\,\boldsymbol{x}_i \tag{5}$$

where

$$\Phi(t) := \begin{bmatrix} 4 - 3c_\Omega & s_\Omega & 0 & 2(1 - c_\Omega) \\ 3s_\Omega & c_\Omega & 0 & 2s_\Omega \\ 6(s_\Omega - \Omega t) & -2(1 - c_\Omega) & 1 & 4s_\Omega - 3\Omega t \\ -6(1 - c_\Omega) & -2s_\Omega & 0 & -3 + 4c_\Omega \end{bmatrix} \tag{6}$$

In Eq. (6), $c_\Omega := \cos(\Omega t)$ and $s_\Omega := \sin(\Omega t)$. Hereafter the subscript i denotes an initial state. Equation (5) is simplified as

$$x(t) = -a\cos(\Omega t + \phi) + \frac{2b}{\Omega} \tag{7}$$

$$y(t) = 2a\sin(\Omega t + \phi) - 3bt + d \tag{8}$$

$$\dot{x}(t) = \Omega a\sin(\Omega t + \phi) \tag{9}$$

$$\dot{y}(t) = 2\Omega a\cos(\Omega t + \phi) - 3b \tag{10}$$

where

$$a := \sqrt{(3x_i + 2\dot{y}_i/\Omega)^2 + (\dot{x}_i/\Omega)^2} \tag{11}$$

$$b := 2\Omega x_i + \dot{y}_i \tag{12}$$

$$d := y_i - 2\dot{x}_i/\Omega \tag{13}$$

$$\phi := \arctan\left(\frac{\dot{x}_i}{\Omega(3x_i + 2\dot{y}_i/\Omega)}\right) \tag{14}$$

Since the follower position forms

$$\left(\frac{x - 2b/\Omega}{a}\right)^2 + \left(\frac{y + 3bt - d}{2a}\right)^2 = 1 \tag{15}$$

the relative motion of the follower represents an elliptic orbit at $b = 0$ and a leader-centered ellipse at $b = d = 0$. Thus the constants a, b, d, and ϕ denote the size of the relative orbit, the drift velocity, the center distance of the ellipse from the leader satellite, and the initial phase, respectively.