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Statistical evaluation of performance impact of manufacturing variability by an adjoint method

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ABSTRACT

Statistical evaluations of the aerodynamic performance changes due to the manufacturing variability for a turbine blade using the first order (FO) and second order (SO) sensitivities are presented in the paper. On the premise that the geometric variation at each scanning point on the blade meets a standard normal distribution, a modeling method taking into account the spatial correlations is used to produce the contour of geometric variations, the basis modes of which are extracted by principal component analysis. Then the geometric variations of any manufactured blades can be described by a series of random variables regarding as the weights of the basis modes. Calculations of the FO and SO sensitivities by using a continuous adjoint method are firstly introduced in detail. The sensitivities of mass flow rate and adiabatic efficiency of the turbine blade to a finite number of primary basis modes are then calculated and sensitivity validations are presented, revealing the significant improvements on performance evaluations by the SO sensitivities. Finally, the statistics of performance changes due to the manufacturing variability are evaluated by Monte Carlo simulations with different probability density functions given for the weights of basis modes. The results demonstrate that by the FO sensitivities, the performance exhibits almost linear changes versus the manufacturing tolerance, while by the SO sensitivities the nonlinear dependence of performance on the geometric variations can be accurately evaluated.

1. Introduction

In the design process for any engineering system, a simplified method by maintaining all the design parameters including the operation conditions, the geometric parameters, et al. as the corresponding nominal ones, is usually employed to make the design process tractable. However, it is well known that the uncertainties of the design parameters always exist in the design process, which cannot be totally eliminated. Since realistic manufacturing processes limit the machining precision, the aerodynamic shape of the final manufactured blade inevitably deviates from the designed one. The spatial distribution of such deviations depends on the machining process. For a number of manufactured blades, such functions of geometric variations follow certain statistical distribution.

Since the middle of last century, it has been well known that the geometric variations dramatically influence the aerodynamic performance of turbocharger blades [1–3]. Since then, lots of studies based on both experimental measurements and numerical simulations have been reported [4–12]. The early studies on performance impact of manufacturing variability were carried out through experimental measurements, by which the effects of the random geometric variations on the performance changes were quantified. However, the experimental cost is extremely high. Due to the rapidly increased computing capabilities and the advanced numerical simulation techniques, computational fluid dynamics (CFD) were used for the studies at the beginning of this century. By the direct Monte Carlo simulations (MCS), Garzon et al. [4] successfully quantified the impact of geometric variations on compressor aerodynamic performance. In the past decade, lots of studies on performance impact of manufacturing variability based on experimental and numerical methods were reported [5–12]. Generally, by a direct CFD method, thousands of samples are necessary for MCS, resulting in unacceptable computing resource requirements, especially for the studies involving three-dimensional flow computations.

In order to reduce the computational cost for the studies of performance impact, the model method rather than the direct CFD method has been employed. For example, the polynomial chaos was employed by Lange et al. [7,8,12] to study the effects of de-

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sign parameter deviations on the performance of a high-pressure compressor stage. By using the model method, a large number of flow computations are required to determine the performance parameters of the training samples. Notice that compared with the geometry scale length of turbomachinery blades, the geometric deviations due to manufacturing variability are sufficiently small. Supposing the sensitivities of performance parameters to geometric variations are all known, the performance changes can be fast calculated by simply summating all the products of geometric variation and the corresponding sensitivity without any additional flow computations [13,14]. The adjoint method, introduced by Jameson [15] in the discipline of aerodynamics, has been widely used for sensitivity calculations due to its high efficiency in calculating the sensitivities. By the adjoint method, the solutions of the governing flow equations and the corresponding adjoint equations each only once are necessary to obtain the complete sensitivities for each performance parameter, regardless of the number of geometric parameters.

The adjoint method was introduced to design optimization of cascades by Dreyer and Martinelli [16], Liu et al. [13,17] at the beginning of this century. In the past decade, the adjoint method has become an active topic and has been widely used in the gradient-based design optimization of turbomachinery blades [18–24]. The present authors have achieved sensitivity calculations by solving the adjoint equations for both the Euler and Navier–Stokes equations [20,21], and used them in the blade design optimization [20–23]. In recent years, the adjoint method was used for performance uncertainty estimations in a quantity of literatures [25–29]. Once the sensitivities are determined, the performance changes can be fast calculated for an arbitrary number of manufactured blades within the tolerance. Giebmanns et al. [25] investigated the performance impact of geometric variations at the leading edge of a two-dimensional blade by using the adjoint method. Xiong et al. [27] and Yang et al. [28] evaluated the performance changes for mass flow rate and adiabatic efficiency of a multi-stage steam turbine by using the adjoint method. It was found that the performance changes linearly depend on the manufacturing tolerance.

Tight manufacturing tolerances result in slight geometric variations, so that the performance parameters exhibit approximately linear variations versus the manufacturing tolerance. However, Garzon et al. [4] demonstrated that the mean performance changes can be decomposed into two parts. The first part is dependent on the averaged geometric variations, while the second part is mainly caused by the nonlinear dependence of performance on the geometric variations. The studies by only the first order (FO) sensitivities [25–28] can successfully capture the first part, while they will fail to capture the second part for the cases with large noise amplitude in the geometric variations. Moreover, tight manufacturing tolerance meaning high manufacturing cost is not always necessary because the aerodynamic performance may not be sensitive to the geometric variations on some portions of the blade. In such cases, loose manufacturing tolerance can be assigned to reduce the manufacturing cost. The performance changes for the manufactured blades with both noise amplitude in the geometric variations and loose tolerance can be evaluated by the second order (SO) sensitivities. Meanwhile, even for tight manufacturing tolerances, evaluations of performance changes by the SO sensitivities can be improved in accuracy. So far, there is no open literature studying the performance impact of manufacturing variability on use of the SO sensitivities.

In the present study, quantifications of geometric uncertainty and the sensitivity-based estimations of performance uncertainty are briefly introduced. A modeling method is adopted to produce the spatial distribution of geometric variations. The quantifications of geometric uncertainty by the modeling method is illustrated for a turbine blade. Calculations of FO and SO sensitivities using a continuous adjoint method are firstly introduced by regarding a finite number of primary basis modes of the geometric variations as the geometric parameters. The adjoint sensitivities and the sensitivity-based performance evaluations are then compared with those obtained from the direct CFD method. The effects of the number of primary basis modes and the manufacturing tolerance on the performance changes are studied. Finally, with respect to different probability density functions given for the weights of basis modes to produce the manufactured blades, the corresponding MCS-based statistical evaluations of the performance changes due to the manufacturing variability are performed. The results are presented and compared in detail.

2. Uncertainty quantification

2.1. Principal Component Analysis (PCA)

The geometric variations of the manufactured blades can be obtained by directly scanning the products. PCA, which is essentially proper orthogonal decomposition (POD) [30] has been successfully used to extract the basis modes of the geometric variations [4,27], by which the most important manufacturing characteristics can be distinguished. On obtaining the scanned geometric variations, a scatter matrix $\mathbf{X} = (x_{ij})_{m\times n}$ can be constructed, where m and n are the numbers of manufactured blades and scanning points for each blade, respectively; x_{ij} is the discrepancy between the local geometric variation and the corresponding average. PCA can then be achieved by decomposing the autocorrelation matrix, $\mathbf{R}$ of the scatter matrix $\mathbf{X}$ by a singular value decomposition (SVD) method.

$$\mathbf{R} = \mathbf{X}^T\mathbf{X} = \mathbf{Q}(\mathbf{\Lambda}^T\mathbf{\Lambda})\mathbf{Q}^T \tag{1}$$

where $\mathbf{Q}$ is the right matrix of eigenvectors of the scatter matrix $\mathbf{X}$; $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues. The basis modes Φ, namely the principal components of the scatter matrix $\mathbf{X}$ are determined by

$$\Phi = \mathbf{\Lambda}\mathbf{Q}^T \tag{2}$$

In principle, for any manufactured blade made from the same machine system, the geometric variations can be described in a weighted summation form of the basis modes.

$$\mathbf{x} = \bar{\mathbf{x}} + \sum_{i=1}^{n} s_i\phi_i \tag{3}$$

where $\mathbf{x}$ is the vector of geometric variations of the manufactured blade; $\bar{\mathbf{x}}$ is the vector of the averaged geometric variations; s_i is the weight of the i-th basis mode; n is the number of basis modes. Mentioned that in any discipline, usually a limit number of primary basis modes take almost the whole energy of the system, meaning that any state vector in the system can be approximately described by a series of primary basis modes. In such cases, only the effects of the primary basis modes on the system performance need to be studied in the eigenvalue problems.

2.2. Modeling of geometry uncertainty

Without a considerable number of manufactured blades, the modeling method used by Schillings et al. [31] and Papadimitriou et al. [32] is employed in the present study to describe the spatial distribution of geometric variations. By the modeling method, the geometric variation of the i-th scanning point is described by a zero-mean random variable with the standard deviation σ_i. Then the zero-mean random vector, $\mathbf{x}_g$ consisted of the random