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Dynamic snap-through of shallow arches under thermal shock

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A B S T R A C T

In the present research, dynamic buckling of a shallow arch subjected to transient type of thermal loading is investigated following the Budiansky–Hutchinson criterion. Arch is subjected to thermal shock on one surface while the other surface is kept at reference temperature. Transient heat conduction equation across the arch thickness is established and solved analytically. The induced bending moment and compressive thermal force are obtained and inserted into the equations of motion of the arch. To model the arch, classical arch theory suitable for thin arches is employed. The von Kármán type of strain-displacement relation suitable for small strains and large deformations is used. The governing nonlinear equations of motion are presented as the coupled nonlinear algebraic equations. These equations are established using the conventional Ritz method, where the shape functions are constructed employing the polynomial functions. The resulting equations are solved by means of the Newmark time marching scheme and the Newton–Raphson linearization technique. By means of the Budiansky criterion, critical thermal shock parameters are distinguished. Dynamic buckling temperatures are also verified using the phase-plane presentation, also known as the Hoff–Hsu criterion. It is verified that the shallow arches may undergo dynamic snap-through type of motion under rapid surface heating when certain geometrical constrains are met.

1. Introduction

The phrase *dynamic buckling* or *dynamic stability* may refer to various complicated motions. For instance, problems dealing with parametric excitation, parametric resonance, and dynamic snap-through fall within this category. As a result, explicit criteria to distinguish the dynamically buckled state of a structure should be defined. One of the most well-known and generally accepted criteria in dynamic buckling analysis is the criterion proposed by Budiansky and Roth which was also extended by Budiansky and Hutchinson [1]. According to this criterion which is also known as equations of motion approach, a structure is buckled in dynamic sense when only an increment in load parameter results in severe change in the structure shape. This approach is one of the most interesting ones in dynamic buckling analysis due to its simplicity in computer programming. However, it may be time consuming. The main advantage of the mentioned criterion is its capability in dynamic buckling prediction for any types of loading, constant or non-constant, with finite or infinite time duration.

The problem of dynamic buckling of shallow arches has a long history. The first investigation, to the best of the present authors' knowledge, belongs to Hoff and Bruce [2]. In the investigation of Hoff and Bruce [2], a half-sine arch is considered which consists an initial deflection. The arch is pinned on both ends and subjected to half-sine distributed lateral load. Two different types of loadings are considered. The case where loading has constant magnitude and infinite duration and the case where loading is similar to an ideal impulse. This investigation motivated many other researchers to explore the response of a shallow arch under sudden mechanical loads of various types.

Mallon et al. [3] performed an investigation on the effects of initial curvature on dynamic buckling loads of shallow arches with different shapes. Results of this study prove that the critical shock parameter may be significantly increased by optimizing the arch shape. Also, the effect of initial imperfection is analyzed. It is also shown that the initial imperfection of the arch has mild effect on the results. Comparison of static and dynamic buckling loads with respect to arch shape, reveal non-trivial quantitative correspondences.

Pi and Bradford [4] analyzed the dynamic snap-through phenomenon in a shallow arch made of an isotropic homogeneous material subjected to the sudden concentrated load at the arch mid-span. In this research, the potential energy criterion, also

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known as the Simitses criterion, is used to distinguish the dynamic buckling load of the arch. The case of the load with infinite duration and constant magnitude is taken into account. It is verified that the static snap-through load serves as an upper bound for dynamic snap-through load when the former one exists. The influence of presence of the static pre-load on the dynamic buckling of a shallow pin-ended arch is also investigated. It is found that the applied static pre-load decreases the dynamic buckling loads, but increases the sum of the applied pre-load and the dynamic buckling load.

Pi et al. [5] investigated the dynamic buckling response of the shallow arch with both ends pinned. The arch is subjected to central concentrated load. A two degrees of freedom arch model is used to establish the energy criteria for dynamic buckling of the conservative systems that have unstable primary and/or secondary equilibrium paths. Afterwards, the energy criteria are applied to the dynamic buckling analysis of fixed shallow arches. It is shown that when the applied load is constant with infinite duration, dynamic buckling loads may be estimated using the Simitses criterion without any need to solve the equations of motion of the arch.

Pi and Bradford [6] performed an investigation on dynamic snapping conditions for a statically preloaded shallow arch accounting for rotational end restrains. The arches are subjected to uniformly distributed radial compression. The potential energy criterion of Simitses is used to distinguish the dynamic buckling load levels. Interestingly, it is shown that arches having equal rotational end restraints may have an upper and a lower dynamic buckling loads, whereas arches having unequal rotational end restraints have only a unique dynamic buckling load.

Using the same formulation, similar to their previous investigations, Pi and Bradford [7] discussed the dynamic snap-through phenomenon in an arch subjected to concentrated mid-span load where the arch is pinned at one end while the other end is built-in. It is revealed that, under a suddenly applied central load, a shallow pinned-fixed arch with a high modified slenderness, has a lower dynamic buckling load and an upper dynamic buckling load, while an arch with a low modified slenderness has a unique dynamic buckling load.

While the dynamic buckling of shallow arches exposed to sudden compressive mechanical loads is well-documented in the open literature, thermally induced dynamic buckling of arches has not been the subject of any studies so far. Meanwhile, thermally induced dynamic buckling of other structures is reported in the open literature.

Shariyat and Eslami [8,9] studied the thermal dynamic buckling of cylindrical shells considering the initial imperfection of the shell. Shell is made of an orthotropic material. Unlike the thin shell formulations which are basically two dimensional, the governing equations of the shell are obtained according to the three dimensional theory of elasticity. The case of shell under sudden uniform temperature elevation is considered in this research. Also, the complete form of the Green strain tensor in polar coordinate system is used. In this work, the Budiansky criterion is implemented to distinguish the thermal buckling loads of the shell in dynamic sense.

Based on the Simitses criterion, Ghiasian et al. [10] analyzed the thermal dynamic buckling of thin Euler–Bernoulli beams made of functionally graded materials. The especial case of a beam with both edges clamped is considered and initial imperfection of the beam is taken into account. Using the Galerkin solution method with polynomial shape functions, the governing equilibrium equations of the beam are solved. The interaction of the beam with a softening nonlinear elastic foundation is also included to generate the unstable equilibrium path for the beam. The load level, where the total modified potential energy of the beams tends to zero, is defined as the dynamic buckling load level of the beam. In this research the thermal load is assumed to be in form of sudden uniform type. It is shown that the slender beam under the sudden thermal shock and resting on a softening elastic foundation is dynamically imperfection sensitive. As a result, with increasing the imperfection parameter of the beam, dynamic buckling temperature alleviates.

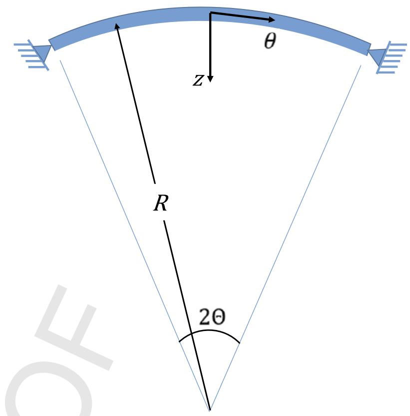

Fig. 1. Schematic and geometric characteristic of a shallow arch.

Ghiasian et al. [11] also performed an investigation on thermal dynamic buckling loads of a shear deformable functionally graded material beam using the Budiansky–Roth criterion. Beams are resting on the sufficiently soft nonlinear elastic foundation to generate an unstable post-buckling branch. The governing equations of motion of the beam are constructed by means of the Ritz method and solved iteratively. It is shown that response of the beam is of the escaping motion type. Therefore, after a certain temperature, midspan deflection of the arch tends to infinity. The phase-portraits of the midspan motion of the beam are also provided to verify the occurrence of dynamic buckling phenomenon. It is shown that at the onset of dynamic buckling, phase portrait of the beam changes drastically.

As mentioned earlier, the large amplitude thermally induced vibrations in a shallow arch is not investigated so far. Therefore, the problem of thermal dynamic buckling of the arch is not also reported. As a first endeavor, present research aims to investigate the dynamic buckling phenomenon of a shallow arch subjected to sudden and rapid surface heating. Arch is made from an isotropic, homogeneous and linearly elastic material. The one dimensional heat conduction equation across the arch thickness is obtained and solved analytically. Afterwards, equations of motion of the arch are established using the Ritz method. The obtained equations which govern the dynamics of the arch under thermal shock are solved for different magnitudes of shock to distinguish the dynamic buckling temperature according to the Budiansky criterion.

2. Fundamental equations

Consider a curved beam made of an isotropic homogeneous material of rectangular cross section $b \times h$ and constant radius of curvature R. Opening angle of the arch is set equal to 2Θ. Therefore, the arch length is $2R\Theta$. Schematic of the arch and the applied coordinates system are shown in Fig. 1. In this system, θ, y, and z represent the circumferential, through the width, and through the thickness directions, respectively.

It is assumed that the shallow arch only deforms in the θ–z plane and therefore deformations through the width are ignored. Displacement field through the beam domain obeys the classical beam theory with the Euler–Bernoulli assumptions which is accurate enough for thin class of beams. Following the slender

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