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Real-time collision avoidance maneuvers for spacecraft proximity operations via discrete-time Hamilton–Jacobi theory

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A B S T R A C T

This study presents a sub-optimal *feedback* control that implements *real-time collision avoidance* for spacecraft in proximity operations. The penalty function for avoiding collision with obstacles is first incorporated into the performance index of a typical optimal tracking problem. Then, the infinite-horizon feedback control law is derived by employing generating functions in the framework of discrete-time Hamilton–Jacobi theory. The derived control law, which is an *explicit* function of the reference states and instantaneous positions of obstacles, allows active spacecraft to avoid collision in real-time. The proposed approach has advantages over conventional optimal collision avoidance approaches in that it does not require iterations with initial guesses, repetitive shooting-based process for multiple boundary conditions, and/or trajectories of obstacles to be known a priori. Numerical simulations demonstrate that the proposed algorithm with a properly designed penalty function is suitable for implementing optimal collision-free transfers in real-time.

1. Introduction

With gradually expanding space industry, spacecraft and their operations have been diversified. Especially, operating multiple small spacecraft has been actively studied as it can implement challenging missions such as 3-D mapping and measurements of the gravitational and magnetic fields with operational efficiency and configurational flexibility. As a key technology for safe operations of multiple spacecraft, various collision avoidance algorithms have been developed. Previous approaches for optimal collision avoidance maneuvers are usually based on direct optimizations [1,2]. They can implement optimal collision avoidance maneuvers by applying inequality constraints, but usually require iterative procedure with initial guesses and trajectories of obstacles to be known a priori. In order to reduce computational loads of direct optimizations, the model predictive control (MPC) has been adapted for optimal collision avoidance maneuvers [3–5]. MPC can reduce computational efforts because it obtains a solution based on the direct optimization over a receding horizon, but it still requires an iterative procedure with an initial guess at every time when obtaining a solution. If the performance of an onboard computer is limited, the indefiniteness due to the initial guess and iterative procedure in the direct optimization approach might tackle real-time implementation. Focusing on the robustness, some studies use gradient of Lyapunov-type function to avoid collision [6–8]. They can apply trajectories of obstacles in real-time without repetitive calculation and initial guess, but they usually do not consider the optimality of collision avoidance maneuvers.

In recent years, the generating function appearing in the Hamilton–Jacobi theory has been used to implement optimal tracking and collision-free transfers in spacecraft formation flying [9–13]. The *continuous-time* generating function was employed to obtain a sub-optimal collision-free trajectory by incorporating the penalty function into the performance index for an optimal transfer [13]. Unlike many indirect methods using the penalty function [14,15], the approach proposed in Lee et al. [13] allows us to derive the sub-optimal feedback control law for finite-time collision-free transfers without iterative procedure and initial guess. However, *it requires trajectories of obstacles to be identified in advance;* the control law obtained from Lee et al. [13] does not work, in general, if the trajectories of obstacles change from the expected one or are not completely known a priori.

Given the above statements, the main technical originality and contribution of this research is summarized as follows: As an alternative method for implementing the optimal collision-free transfer in real-time, this study proposes to employ the recently developed infinite-horizon optimal tracking scheme in the framework of *discrete-time* Hamilton–Jacobi theory [12]. Assigning the reference

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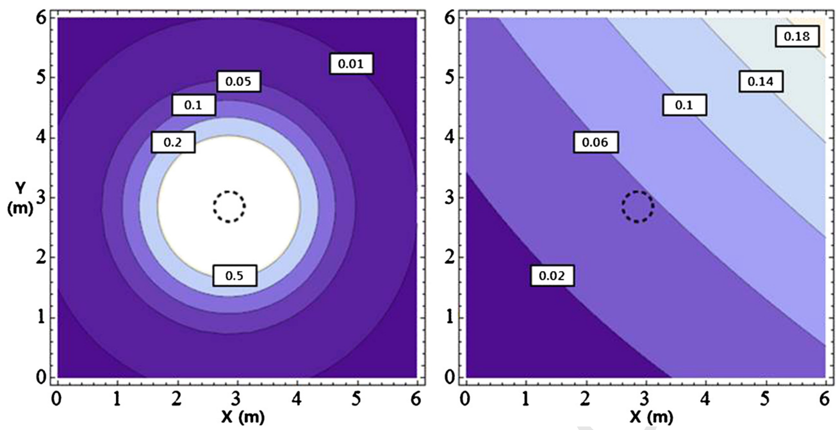

Fig. 1. Contours of the original penalty function (left) and approximated penalty function by the Taylor series expansion about a zero nominal solution (right).

states to independent variables, this scheme can derive an infinite-horizon optimal tracking control law as an explicit form of the reference states. Thus, incorporating the penalty terms for avoiding collision with obstacles into the tracking performance index, and assigning the reference states and positions of obstacles to independent variables, we derive the feedback control law for optimal collision-free transfers as an *explicit function* of the reference states and positions of obstacles. Compared with conventional optimal collision avoidance approaches, the proposed approach does *NOT* require *iterations* with initial guesses, *repetitive* shooting-based process for multiple boundary conditions, and/or trajectories of obstacles to be *known a priori*.

The rest of this paper is organized as follows. The sub-optimal collision avoidance problem is first formulated in discrete-time domain. Then, the procedure for deriving the infinite-horizon control law for optimal collision-free transfers is constructed. Simulations follow for the control law to demonstrate the validity of the proposed approach. Finally, conclusions are drawn.

2. Infinite-horizon optimal collision-free control law employing discrete-time generating functions

The optimal collision-free transfer with continuous-thrust control can be formulated in continuous-time domain as follows:

Minimize

$$J = \frac{1}{2}\int_{t_0}^{t_f} \mathbf{u}^T\mathbf{u}dt \tag{1}$$

subject to general nonlinear equations of motion in affine form with rendezvous-type boundary conditions and inequality constraints for circumventing obstacles

$$\begin{aligned}\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},t) + \mathbf{g}(\mathbf{x},t)\mathbf{u}, \quad \mathbf{x}(t_0) = \mathbf{x}_0, \quad \mathbf{x}(t_f) = \mathbf{x}_f, \\ \left\|\mathbf{r} - \mathbf{r}_{o,i}\right\| \geq d_{\min,i}, \quad i = 1, \cdots, s\end{aligned} \tag{2}$$

where $\mathbf{x} \in \mathbf{R}^{n\times 1}$ and $\mathbf{u} \in \mathbf{R}^{m\times 1}$ are the state and control vectors of active spacecraft, respectively, and $\mathbf{r}$ and $\mathbf{r}_{o,i}$ are the position vectors of active spacecraft and the i-th obstacle, respectively. s is the number of obstacles, and $d_{\min,i}$ is the desired minimum relative distance from the center of the i-th obstacle. To implement a collision-free transfer in real-time based on the infinite-horizon control scheme proposed in [12], the above problem is reformulated as an optimal tracking problem in *discrete-time* domain as follows:

Minimize

$$\begin{aligned}J &= \frac{1}{2}(\mathbf{x}_N - \mathbf{x}_r)^T Q\,(\mathbf{x}_N - \mathbf{x}_r) \\ &+ \sum_{k=0}^{N-1}\left[\frac{1}{2}(\mathbf{x}_k - \mathbf{x}_r)^T Q\,(\mathbf{x}_k - \mathbf{x}_r) + \mathbf{u}_k^T R\mathbf{u}_k + P\right]\end{aligned} \tag{3}$$

subject to discrete-time nonlinear equations of motion in affine form with initial conditions

$$\mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, t_k) + \mathbf{g}_d(\mathbf{x}_k, t_k)\mathbf{u}_k, \quad \mathbf{x}_0 \text{ is given} \tag{4}$$

where t_k and $\mathbf{x}_k$ are time and states at the k-th step, respectively, and $Q \in \mathbf{R}^{n\times n}$ and $R \in \mathbf{R}^{m\times m}$ are weighting matrices. $\mathbf{f}_d$ and $\mathbf{g}_d$ are obtained by discretizing $\mathbf{f}$ and $\mathbf{g}$, respectively. $\mathbf{x}_r$ is the *reference states, which is defined as the unconstrained optimal solution minimizing Eq. (1) without the inequality constraints*. P is the penalty function to indirectly assign the forbidden region for collision-free maneuvers.

The above discrete-time optimal tracking problem can be transformed into a two-point boundary value problem of states and costates for a discrete-time Hamiltonian system [11,16]. Then, the infinite-horizon control law can be derived by employing the discrete-time generating function of the second type, F_2 [12]. F_2 can be obtained by recursively solving the discrete-time Hamilton–Jacobi equation after the Hamiltonian is approximated as a Taylor series about a nominal solution. (Again, no repetitive or iterative process lies in developing the F_2 generating functions!) In the previous studies employing the generating functions, the nominal solution is usually defined as an equilibrium solution such as the origin of the coordinates for a double integrator. However, *fixing the nominal solution as an equilibrium solution might lead to undesired poor approximation and thus fail to avoid collision*; the series expansion of (highly nonlinear) penalty function sufficiently approximates the original penalty function *only quite near* the nominal solution. As an example, Fig. 1 shows the contours of the approximated penalty function by Taylor expansion, where the original penalty function is defined as $1/\|\mathbf{r} - \mathbf{r}_o\|^4$ with $\mathbf{r}_o = [2.85, 2.85]^T$ m. The approximated penalty function about a zero nominal solution is expanded as Taylor series up to the second order as follows:

$$\begin{aligned}&\frac{4x_o}{\|\mathbf{r}_o\|^6}x + \frac{4y_o}{\|\mathbf{r}_o\|^6}y + \frac{24x_o y_o}{\|\mathbf{r}_o\|^8}xy \\ &+ \frac{1}{2}x^2\left(\frac{24x_o^2}{\|\mathbf{r}_o\|^8} - \frac{4}{\|\mathbf{r}_o\|^6}\right) + \frac{1}{2}y^2\left(\frac{24y_o^2}{\|\mathbf{r}_o\|^8} - \frac{4}{\|\mathbf{r}_o\|^6}\right)\end{aligned} \tag{5}$$

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