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Robust and global attitude stabilization of magnetically actuated spacecraft through sliding mode

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ABSTRACT

The inertial pointing problem of a rigid satellite by solely magnetic torqueing is considered in this paper. To ensure globally uniformly ultimately bounded motion about the reference in inertial space, a sliding mode attitude control law, which consists of equivalent and reaching control terms, based on a novel time-varying sliding manifold is designed. The originality of the sliding manifold relies on the inclusion of two time-integral terms. The usage of the proposed sliding manifold makes the application of the equivalent control method to the considered problem possible, and it is proven that the state trajectories reach the newly designed sliding manifold in finite time even under the effect of four realistically modeled disturbance components and parametric uncertainty of all inertia matrix entries. For the constructed purely magnetic attitude control system, stability and existence of the sliding mode as well as state trajectories' finite time convergence to the sliding manifold are demonstrated via Lyapunov function techniques. The results of a simulation example verify the robust stability of the designed attitude control system. The steady state performance of the attitude control system is evaluated in the altitude range of low-Earth-orbits.

1. Introduction

SPACECRAFT'S angular motion around their center of gravity, namely their attitude is controlled around their three body axes basically by onboard torque producing actuator triads. The triads consist of three orthogonally placed actuators. There are three types of such actuators: momentum exchange devices (reaction/momentum wheels, control moment gyroscopes), reaction thrusters (gas jets), and magnetic torquers (rods, coils). The members of the first two types produce directly the control torques around the body axis along which they are placed on the spacecraft. The loss of any member renders the attitude control system underactuated around that axis. The asymptotic stabilization problem of such directly torque producing and underactuated systems have been an important topic of investigation for researchers in automatic control field [1–3]. The conclusion is that it is impossible to stabilize such underactuated rigid spacecraft in even locally asymptotical manner by using continuous time-invariant control laws [4].

The sole usage of the third type of actuators leads to a challenging and therefore interesting control problem. Even if all three magnetic torquers are operational, the control torque lies in a plane, which is orthogonal to the geomagnetic field vector at the satellite's location. This phenomenon emerges from the fact that, according to physics, the magnetic control torque is the output of the cross-product of the magnetic moment vector, which is what the magnetic actuator triad directly produces, with the local geomagnetic field vector. Thus, attitude control by purely magnetic actuation lacks three-axis control authority intrinsically. However, the controllability of such an underactuated system could be proven thanks to its second challenging property, time-variance, by using a nonrotating dipole model for the geomagnetic field [5]. The time-variance results from the orbital motion of the satellite around the Earth provided that the orbital plane does not coincide with the equatorial plane of the geomagnetic field. As a result, the system can be considered to be *instantaneously underactuated* [6] because the direction along which there is no control

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Nomenclature

$\vec{B}$	local geomagnetic field vector of size 3×1 [T]
b	control matrix (7×3)
b_n	nominal control matrix (7×3)
C_B	projection matrix onto the plane orthogonal to $\vec{B}$ (3×3) [–]
D_B	projection matrix onto the direction along $\vec{B}$ (3×3) [–]
$\vec{d}$	total disturbance vector (7×1)
$\vec{d}_n$	nominal disturbance vector (7×1)
$\vec{d}_{unc}$	disturbance vector solely due to satellite model uncertainty (7×1)
$\vec{f}_n$	nominal system vector (7×1)
h	orbital altitude [km]
I	identity matrix (3×3) [–]
J	uncertain inertia matrix (3×3) [kg m^2]
J_i	principal moments of inertia [kg m^2], $i = 1, 2, 3$
J_n	nominal inertia matrix (3×3) [kg m^2]
$k_{\text{int}\,q}$	dimensionless sliding surface design parameter [–]
k_q	sliding surface design parameter [rad/s]
k_s	continuous reaching law design parameter [N ms]
k_{ss}	discontinuous reaching law design parameter [N m]
$\vec{M}$	magnetic control moment vector (3×1) [A m^2]
m	input number of the control system [–]
n	orbital angular velocity (mean motion) [rad/s], order of the control system [–]
$\vec{q}$	vectorial quaternion component (3×1) [–]
$\vec{q}_{\perp\vec{B}}$	$\vec{q}$'s component orthogonal to $\vec{B}$ [–]
$\vec{q}_{\parallel\vec{B}}$	$\vec{q}$'s component along $\vec{B}$ [–]
$\tilde{q}$	skew-symmetric matrix of the vectorial quaternion component (3×3) [–]
q_4	scalar quaternion component [–]
$\vec{s}$	sliding surface vector (3×1) [rad/s]
T	orbital period [s]
$\vec{T}_{aero}$	aerodynamic drag torque vector (3×1) [N m]
$\vec{T}_d$	environmental disturbance torque vector (3×1) [N m]
$\vec{T}_{gg}$	gravity-gradient torque vector (3×1) [N m]
$\vec{T}_{mag}$	residual magnetic torque vector (3×1) [N m]
$\vec{T}_{mc}$	magnetic control torque vector (3×1) [N m]
$\vec{T}_{solar}$	solar pressure torque vector (3×1) [N m]
$\vec{T}_{unc}$	disturbance torque vector solely due to satellite model uncertainty (3×1) [N m]
t_s	starting moment of the sliding mode [s]
t_0	starting moment of the control process [s]
$\vec{u}$	control vector (3×1) [N m]
$\vec{u}_{eq}$	equivalent control vector (3×1) [N m]
$\vec{u}_{reach}$	reaching control vector (3×1) [N m]
$\vec{u}_{\parallel\vec{B}}$	$\vec{u}$'s component parallel to $\vec{B}$ [N m]
$\vec{u}_{\perp\vec{B}}$	$\vec{u}$'s component orthogonal to $\vec{B}$ [N m]
$\vec{\omega}$	absolute angular velocity vector (3×1) [rad/s]
$\vec{\omega}_{\perp\vec{B}}$	$\vec{\omega}$'s component orthogonal to $\vec{B}$ [rad/s]
$\vec{\omega}_{\parallel\vec{B}}$	$\vec{\omega}$'s component along $\vec{B}$ [rad/s]
$\tilde{\omega}$	skew-symmetric matrix of the absolute angular velocity vector (3×3) [rad/s]
$\vec{x}$	state vector (7×1)
$\vec{x}_N$	reference state vector for inertial pointing (7×1)
ΔJ	inertia uncertainty matrix (3×3) [kg m^2]
$\vec{\gamma}$	auxiliary torque vector (3×1) [N m]
θ	attitude (Euler) angle [deg]
λ	eigenvalue
φ	attitude (Euler) angle [deg]
ψ	attitude (Euler) angle [deg]
$\|..\|_2$	L_2 (quadratic) norm of a vectorial signal
$\|..\|_\infty$	L_∞ norm of a vectorial signal
$|..|$	determinant of a matrix
$\|..\|_{i2}$	induced L_2 norm of a matrix

authority varies with respect to the body axes while the satellite moves along its orbit. The controllability of attitude control systems employing gas jets [7] and reaction wheels [8] has also been investigated before for both the cases of full and lacking control authority.

Since magnetic actuators suit the small satellite concept well regarding their favorable properties in terms of mass, volume, nominal power consumption, low failure risk due to nonmoving structural elements, the research interest in purely magnetic attitude control problem also serves purposes of engineering application. If the satellite's mission requires moderate pointing accuracy ($>1°$), no rapid stabilization and agile maneuvering (in hours), magnetic actuators have the capability to serve as primary actuators. Besides many university pico/nanosatellites (mostly CubeSats) controlled by purely magnetic actuation [9], ORBCOMM [10], and Ørsted [11] are two microsatellites actively controlled by solely magnetic means in addition to passive gravity gradient stabilization assist. A remarkable application of this approach is the GOCE mission by ESA. The satellite GOCE weighing 1052 kg, which has a passively aerodynamically stable structural design, employed only a magnetic rod triad for attitude control and is the first and so far only nonsmall satellite with such an attitude control system [12]. A recently launched minisatellite Proba-V of ESA, which has no passive attitude stabilization assist, utilizes purely magnetic three-axis stabilization in its safe mode [13]. These real life examples indicate the industrial need for control algorithms that will drive a magnetic torquer triad in more beneficial ways, especially in ways guaranteeing global and robust stabilization without passive stabilization assist.

It is aimed with this paper to present in detail an achievement in global and robust attitude stabilization of a rigid satellite by purely magnetic actuation in a nearly circular orbit with low altitude, which does not lie in the geomagnetic equatorial plane; this achievement has been first presented briefly in [14]. The majority of works in literature dealing with purely magnetic attitude control in three-axis proposed local solutions [15–23]. In [20], which is the first and only literature survey on purely magnetic attitude control, particularly local approaches to the problem are well classified. [23] proposes a robust, but local solution to the problem by designing a control system that has stability robustness against model uncertainty via periodic-state feedback and H_∞ control. There is a limited number of works that propose global solutions to the considered problem, which can be summarized as follows:

1) A globally asymptotical solution to the Earth (nadir)-pointing problem; valid for satellites with gravity-gradiently stable inertia distribution, based on the periodicity assumption of the geomagnetic field [24,25]. The periodic extension of the Lyapunov's stability theory is used to derive the state-feedback controller.
2) An almost globally asymptotical solution to the Earth-pointing problem; valid for satellites with gravity-gradiently stable inertia distribution, based on average controllability of the system provided by the – not necessarily periodic – variation of the geomagnetic field during one orbital period [26]. The averag-

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