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Near optimal finite-time terminal controllers for space trajectories via SDRE-based approach using dynamic programming

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ABSTRACT

This paper presents a novel development to synthesize finite-time near optimal feedback control for nonlinear systems with nonlinear terminal constraints such as hypersurfaces. Especially when terminal hypersurfaces are posed as transcendental equations, the developed SDRE-based method contributes first-ever treatment for such cases. The SDRE-based approach, to synthesize continuous-time terminal controllers, is first extended for the fixed-final-time optimal control problems via solving the pointwise governing Hamilton–Jacobi–Bellman equation subject to the pseudo-linear dynamical system with linear terminal constraints. Then, to fit these derived settings into a general class of terminal constraints as hypersurfaces, the method of successive linearization is employed to obtain approximated hyperplane which facilitates state-dependent boundary conditions in order to compute the feedback control input. To establish the developed methodology, numerical investigations on nonlinear systems including the fixed-finite-time optimal control problem of spacecraft spin maneuvers with a variety of terminal cases are illustrated with details. The obtained feedback solution, for all the examples, is compared with the respective openloop solution to validate the efficacy of the novel approach that accomplishes a very high accuracy of the synthesized terminal controller incurring the least cost-to-go even though terminal hypersurface has multiple endpoint solutions which are not a priori known.

1. Introduction

Improving on control design for real-world applications has always been under investigation to ensure optimized performance. Especially in the context of systems' performance, to meet the boundary conditions with stringent precision, terminal controllers draw a special interest in many scientific fields [1]. Of particular interest are the problems that focus on engineering applications, which also can be cast with the cost requirements. In particular, in the field of aerospace, there are many challenging problems such as optimal large-angle maneuvers of spacecraft, multibody trajectory planning, space robotics, orbital station-keeping, spacecraft formation flying, rendezvous and docking missions, guidance of multi-agent co-operative systems, control of unmanned aerial vehicles (UAVs), collision avoidance, etc., which clearly require precise terminal control solutions for attaining the best performance. In the pursuit of dealing with such problems, if the mathematical modeling is assumed as linear system, a vast literature on theoretical and practical developments is readily available for controller's design and analyses. But, considering these problems remains extremely daunting when the system's dynamics and boundary constraints are nonlinear.

Apropos modern advancements for handling nonlinear models, in practice there is no unified approach which can adequately accomplish fast and optimized control solutions accommodating all types of nonlinearities and unknown disturbances, the subject of optimal control, nevertheless, has been well-practiced to enable technical solutions in many areas including significant developments toward aerospace applications since the past few decades. Optimal control theory [2] offers foundational background to solve both discrete-time and continuous-time optimal control problems (OCPs) in openloop and feedback fashion. To yield openloop optimal solutions, based on the classical calculus of variations approach [3], the first-order necessary conditions for optimality transforms the OCP into a two-point boundary value problem (TPBVP), which can be solved by using direct and/or indirect iterative techniques [4]. As noted, the available regime of these techniques heavily depends upon judicious initial guesses to converge at a candidate solution which must be tested further with the second order conditions to be the optimal. As such control design via openloop methods is not considered very practical due

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to the method's susceptibility to the initial conditions, time-to-go, and disturbances; however, these solutions, if available, are often utilized to learn the systems' forced-behavior and accepted as the benchmark to compare the same with the counterpart in the feedback form; it can also be useful as a nominal solution to construct neighboring optimal control [2].

Another theoretical approach in optimal control, which is quite dedicated to obtain feedback extremals [7], is established as dynamic programming [5–7]. In the context of OCPs, this approach mainly proceeds with formulating the governing Hamilton–Jacobi–Bellman (HJB) equation [2] which is basically a scalar first-order nonlinear partial differential equation (PDE). Solution of this PDE categorically attains optimal feedback control without checking the second order conditions for local optimality. However, obtaining analytical solution of the HJB equation, even for a low dimensional linear system, is a formidable task; hence, to solve it numerically, the available techniques as in [8–12] and many others mostly rely on algorithms in-built from qualified approximations, which, in general, much depends upon systems' nonlinearities, terminal constraints and control bounds. Therefore, due to major difficulties in approximating the solution of the constrained finite-time HJB equation, much of this kind of framework is displayed on infinite-time OCPs; the finite-time OCPs with nonlinear terminal constraints have not received its due attention.

Focusing on the literature review for terminally constrained OCPs, these problems are mostly viewed as special cases in terms of soft or point terminal constraints. To approximate the control solution with respect to point terminal constraints, Caetano and Yoneyama [13] developed an iterative scheme using linear-quadratic (LQ) approximation. Wei et al. [14] studied another iterative method derived from continuous-time differential dynamic programming to solve finite-time OCPs with point terminal constraints. In [15,16], Park and Scheeres presented a generating function approach that uses canonical transformations to compute the closed-form solution for general boundary conditions. However, in all of these case-by-case developments, there is no formal discussion of nonlinear terminal constraints as such. To address it first time, by using the dynamic programming approach, Vadali and Sharma [17] devised the series solution methodology (SSM) for generating higher-order optimal feedback control for a class of nonlinear systems with nonlinear terminal constraints. Especially for the LQ terminal controller, the SSM is presented as an alternative of two-step sweep method [2] to derive the closed-form solution. To broaden the capabilities of SSM, Sharma et al. [18] extended the series-based technique using a waypoint scheme. The SSM, as established in [17,18], achieves terminal controller for nonlinear systems, however, the methodology is confined because of its governing structure based on Taylor's series expansion of the HJB equation given in the polynomial form. To deal with a non-polynomial high-dimensional (3 or more) system, the SSM relatively requires more terms in the approximating series, which sometimes may encounter curse of dimensionality [5] resulting in excessive computational burden of gain tensors to influence the control input and the desired terminal accuracy. In the same direction, to handle discrete-time terminally constrained OCPs up to the level of terminal hypersurface, Heydari and Balakrishnan [19,20] explored the method of adaptive critics to obtain the feedback neuro-controller. The established controller, however, is precisely an outcome of an iterative approach that, with some mild conditions, much depends on the judicious offline training to obtain the neural-network gains a priori. Using approximate dynamic programming, another seemingly fast iterative approach named as model predictive static programming was contributed by Padhi and Kothari [21] to present suboptimal solution which is applicable for discrete-time nonlinear systems only. Also, the method uses small error approximation that requires good initial guess history of control solution to initiate the iterative algorithm for point constraints only. Maity et al. [22] recently proposed an extended and computationally efficient version of [21] as generalized model predictive static programming given in continuous time, however, it was with the same limitations as in [21].

In the perspective of some other noteworthy approaches to terminally constrained OCPs using the dynamic programming formalism, Heydari and Balakrishnan [23,24] recently contributed a versatile method of using the finite State-Dependent Riccati Equation (SDRE) method to synthesize suboptimal closed-form solution of the fixed-finite-time OCPs, but the work was just limited with soft terminal constraints only. Parsley and Sharma [25] presented near optimal finite-time feedback guidance for lunar landing problem by extending the SDRE-based technique using the backward sweep method [2], however the solution procedure is examined for soft and point terminal constraints only. Steinfeldt and Tsiotras [26] studied the use of HJB framework with the SDRE-technique for the infinite-time regulation problem by using sum-of-squares programming to investigate the closed-loop stability and robustness.

In this paper too, motivated from the design of LQ terminal controllers and the prominence of SDRE-based practical techniques, the SDRE-type formalism is utilized to develop the new methodology in the feedback form. Briefly, just to trace out the usage of the SDRE technique for control applications, first time, Cloutier et al. [27–30] presented this technique to deal with infinite-time OCPs and its applications to aerospace systems. In order to generate the feasible feedback control input for nonlinear system, the SDRE technique fundamentally exploits pointwise fusion of LQ-type control design by expressing the nonlinear system as a pseudo-linear system at every time point in computation. As elaborated in the recent survey paper on the SDRE by Cimen [31], the SDRE-based controllers are widely implemented in the bulk of applications in many different areas including complex aerospace [31] and nonlinear analytic systems [32] due to its attractive stability domain, robustness, low computational needs and effectiveness to deliver result-oriented suboptimal solution [32–34]. Furthermore, details on the SDRE techniques can be viewed in [31–34] and the references therein.

As just discussed above, to the best of authors' knowledge, the literature thus far delineates no such direct treatment to deal with general fixed-finite-time OCPs with generalized hard constraints such as terminal manifolds/hypersurfaces in the feedback formalism. Specifically, for the OCP in which the terminal constraint is prescribed as a transcendental equation or if it is not prespecified as a unique point, there is no available technique to determine the optimal feedback solution just because such OCPs become extremely challenging to conduct further investigations about checking optimality conditions. To step ahead in this direction, this paper clearly presents near optimal nonlinear feedback synthesis of terminal controllers by obtaining a novel extension of the SDRE-based technique for continuous finite-time OCPs using the dynamic programming approach, which is proven highly effective to facilitate a unified solution procedure to deal with a large class of high-dimensional nonlinear systems given with general boundary conditions as point, hyperplane or hypersurface.

The findings in this paper are organized as follows: Section 2 describes the mathematical formulation of the OCP with general terminal constraints. Then, Section 3 proceeds with the theoretical development of the novel SDRE-based method using the dynamic programming approach. Next, to show the efficacy of the methodology, Section 4 thoroughly discusses numerical illustrations of three distinct examples spanning from one-dimensional nonlinear system to three-dimensional finite-time spacecraft spin maneuver problem with a variety of cases. Finally, Section 5 presents conclusions with remarks followed by the list of key references.