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Near time-optimal feedback instantaneous impact point (IIP) guidance law for rocket

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A B S T R A C T

This paper proposes a feedback guidance law to move the instantaneous impact point (IIP) of a rocket to a desired location. Analytic expressions relating the time derivatives of an IIP with the external acceleration of the rocket are introduced. A near time-optimal feedback-form guidance law to determine the direction of the acceleration for guiding the IIP is developed using the derivative expressions. The effectiveness of the proposed guidance law, in comparison with the results of open-loop trajectory optimization, was demonstrated through IIP pointing case studies.

1. Introduction

The instantaneous impact point (IIP) of a rocket, given its position and velocity, is defined as its touchdown point assuming a free-fall flight (without propulsion) [1]. The IIP is considered as a very important information for safe launch operation of a rocket, and it should be calculated and monitored in real-time on the ground facility or on-board of the rocket. The trespassing of IIP trajectory across a destruction line (DL) is one important criterion for a range safety decision – activation of the flight termination system (FTS) – for flight safety operation. A number of studies on prediction of the IIP and their applications for flight safety operation could be found in the literature. These studies include the techniques for computing the IIP in various coordinate systems [2–5], methods on compensation for the effects of gravity perturbation and atmospheric drag [6], expressions for the time derivatives of IIP [7], and introduction of a new flight safety criterion [8].

In addition to flight safety operations, the IIP can be used for pre-flight analysis and open-loop optimization of a rocket, particularly to obtain and specify the impact point of separated stages. Yoon and Ahn proposed a trajectory optimization procedure considering the IIPs of the first-stage and payload fairing segments of a launch vehicle as explicit constraints [9]. Using the dispersion analysis, Mandic introduced a guidance and control algorithm that can steer a rocket so that its impact point reaches a target location [10]. The IIP change is important for recent landing guidance of a separated stage of a reusable launch vehicle. For example, it is known that the landing guidance for the separated first stage of Falcon 9 involves the “boostback burn” using three out of nine engines, which change the IIP of stage toward the landing site (barge ship or launch site) [11].

This paper proposes a new near time-optimal feedback guidance law that moves the IIP of a rocket to a target point, whose schematic diagram is shown in Fig. 1. The analytic formulation that describes the time derivatives of an IIP for a given external acceleration vector (primarily produced by the propulsion system) was established. An optimization problem that determines the components of the external acceleration vector to align the IIP derivative vector with the desired direction and maximize its magnitude was formulated, and it can be solved analytically by introducing the Lagrange multipliers. The proposed guidance law was validated through a case study and compared with the results of an open-loop trajectory optimization to minimize the final time.

Three key contributions of this study are summarized as follows. First, the proposed guidance law is a feedback form that can explicitly specify the IIP at the final time. Since the guidance law is a feedback form, it is robust to the error coming from various sources (e.g., the position and velocity errors at the beginning of the guidance). Second, its performance is near time-optimal, and near fuel-optimal assuming the acceleration profile of the rocket is given. Lastly, the proposed law does not involve any iterative procedure, which is a very attractive property for its potential on-board implementation.

This remainder of this paper is organized as follows. Section 2 introduces the methodology to calculate the Keplerian IIP and its

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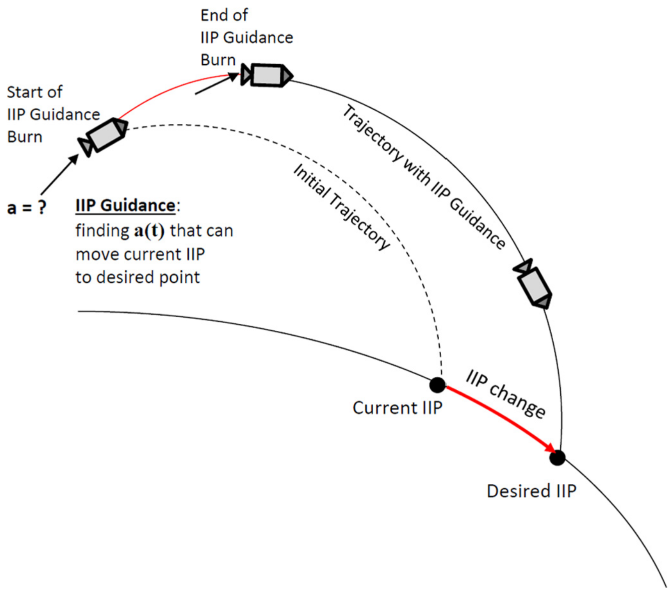

Fig. 1. Schematic diagram explaining IIP guidance.

time derivatives. Section 3 proposes a feedback guidance law to move the IIP of a rocket to a desired point in a near time-optimal manner, which is obtainable by solving a constrained optimization problem established based on the results of Section 2. Case studies for validating the guidance law are presented in Section 4. Finally, Section 5 discusses the comprehensive conclusions of this study and potential opportunities for future work.

2. Calculation of IIP and its time derivatives

This section introduces the procedures to calculate the IIP and its time derivatives in an inertial (Earth centered inertial, ECI) and rotating (Earth centered Earth fixed, ECEF) frames, which provides the fundamentals of the feedback IIP guidance law discussed in this paper. Note that Subsections 2.1 and 2.2 are written by summarizing the results of prior studies conducted by Ahn and Roh [5, 7]. The parameters and geometry used to compute the IIP and its time derivatives are shown in Fig. 2.

2.1. Calculation of Keplerian IIP [5]

Consider the translational motion of a rocket subject to gravity ($\mathbf{g}$) and an external acceleration ($\mathbf{a}$) as follows:

$$\dot{\mathbf{r}} = \mathbf{v} \tag{1}$$

$$\dot{\mathbf{v}} = \mathbf{g} + \mathbf{a} = \mathbf{g}(\mathbf{r}) + a_r \mathbf{i}_r + a_\theta \mathbf{i}_\theta + a_h \mathbf{i}_h \tag{2}$$

In the dynamic equations, $\mathbf{r}$ and $\mathbf{v}$ are position and velocity of the rocket, respectively; a_r, a_θ, and a_h are the components of acceleration vector in position, tangential, and linear momentum directions, respectively, and $\mathbf{i}_r$, $\mathbf{i}_\theta$, and $\mathbf{i}_h$ are the unit vectors of the directions. If the Keplerian two-body motion is assumed, the gravitational acceleration is expressed as

$$\mathbf{g}(\mathbf{r}) = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} \tag{3}$$

Given current position ($\mathbf{r}_0$) and velocity ($\mathbf{v}_0$) of the rocket, its IIP in the ECI coordinate frame is expressed as follows:

$$\mathbf{i}_p = \frac{\cos(\gamma_0 + \phi)}{\cos\gamma_0}\mathbf{i}_{r0} + \frac{\sin\phi}{\cos\gamma_0}\mathbf{i}_{v0} \tag{4}$$

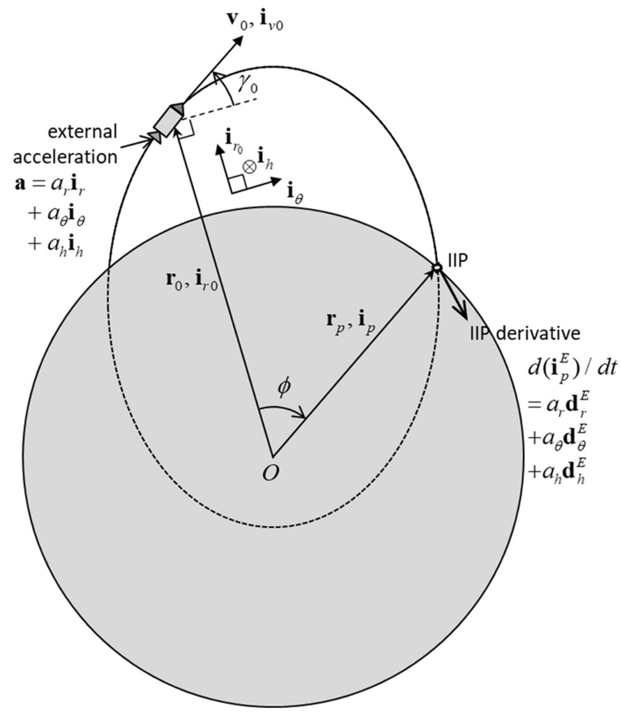

Fig. 2. Parameters and geometry for computing IIP and its derivatives.

In this equation, γ_0 and ϕ are respectively the flight path angle and the angle of flight of the rocket expressed as

$$\gamma_0 = \sin^{-1}\left(\frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\|\mathbf{r}_0\|\|\mathbf{v}_0\|}\right) = \sin^{-1}\left(\frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{r_0 v_0}\right) \tag{5}$$

$$\phi = \sin^{-1}\left(\frac{b_1 b_3 + \sqrt{b_1^2 b_3^2 - (b_1^2 + b_2^2)(b_3^2 - b_2^2)}}{b_1^2 + b_2^2}\right) \tag{6}$$

where b_1, b_2, and b_3 are expressed as

$$b_1 = -\frac{h}{\mu r_0}(\mathbf{r}_0 \cdot \mathbf{v}_0), \qquad b_2 = \frac{h^2}{\mu r_0} - 1, \qquad b_3 = \frac{h^2}{\mu r_p} - 1 \tag{7}$$

The time of flight of the launch vehicle – between the current time and the impact time – is expressed as follows [12]

$$t_F = \frac{r_0}{v_0 \cos\gamma_0}\left(\frac{\tan\gamma_0(1-\cos\phi) + (1-\Lambda)\sin\phi}{(2-\Lambda)\left(\frac{1-\cos\phi}{\Lambda\cos^2\gamma_0} + \frac{\cos(\gamma_0+\phi)}{\cos\gamma_0}\right)} + \frac{2\cos\gamma_0}{\Lambda(\frac{2}{\Lambda}-1)^{1.5}}\tan^{-1}\left(\frac{\sqrt{\frac{2}{\Lambda}-1}}{\cos\gamma_0\cot(\frac{\phi}{2}) - \sin\gamma_0}\right)\right) \tag{8}$$

where Λ $(\equiv (v_0/v_c)^2 = r_0 v_0^2/\mu)$ is defined as the square of the ratio between the current velocity and the circular orbit velocity with given radius ($v_c = \sqrt{\mu/r_0}$). The IIP latitude and longitude in the ECI coordinate system can be expressed using the components of the IIP unit vector in Eq. (4) as

$$\mathrm{Lat}_p = \sin^{-1}(i_{pz}) \tag{9}$$

$$\mathrm{Lon}_p = \arctan^2(i_{py}, i_{px}) \tag{10}$$

The IIP longitude in the ECEF coordinate system is obtained by reflecting the Earth's rotation during the time of flight as

$$\mathrm{Lon}_p^E = \mathrm{Lon}_p - \omega_e(t - t_{ref} + t_F) = \mathrm{Lon}_p - \omega_e \Delta t \tag{11}$$