Aerospace Science and Technology 75 (2018) 284–296

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Distributed almost global finite-time attitude consensus of multiple spacecraft without velocity measurements

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ARTICLE INFO

Article history:
Received 23 September 2016
Received in revised form 3 November 2017
Accepted 8 December 2017
Available online 3 February 2018

Keywords:
Attitude consensus
Finite-time control
Output feedback
Input saturation
Unwinding

ABSTRACT

This paper addresses the attitude consensus problem of multiple rigid bodies in terms of the unit quaternion parameterization. By employing Lyapunov theory and homogeneous techniques, distributed finite-time attitude consensus laws are proposed for leader-following and leaderless multi-agent systems, with full-state (i.e., attitude plus angular velocity) or attitude-only measurements. Specifically, sliding mode observers are used to estimate the leader's information in finite time for followers without direct access to the leader. The so-called "separation principle" is then established between the observers and the consensus controllers. In addition, quaternion filtering systems are constructed to inject the necessary damping into the closed-loop system when angular velocity measurements are absent. In all scenarios, the proposed methods ensure almost global finite-time convergence, avoid the unwinding problem, and yield continuous control torques with *a priori* known bounds. Numerical examples demonstrate the effectiveness of the proposed methods.

1. Introduction

Cooperative attitude consensus of a team of rigid bodies has attracted great interests recently due to its broad applications in spacecraft formations, networked unmanned aerial or underwater vehicles, etc. The problem is very challenging because the attitude configuration SO(3), the set of 3×3 rotation matrices, is a compact non-contractible manifold, and the attitude kinematics and dynamics are both nonlinear. It is more complicated than the attitude control for a single rigid body because certain collective behavior and information flow are required among the team members. Generally, attitude consensus control can be categorized into leaderless consensus [1,2], cooperative tracking [3–13], and containment control [14], where the attitudes of the team members reach a synchronized state, track a leader's trajectory, and enter a convex hull of multiple leaders' attitudes, respectively. This paper mainly focuses on the former two types.

Attitude synchronization and/or tracking laws for the above approaches were designed either directly on SO(3), or in terms of modified Rodrigues parameters (MRPs), a minimal yet non-global parameterization, or unit quaternions, a global redundant parameterization. For the leader-following consensus problem, the methods of [3–8] necessitate that every follower access the leader's trajectory information, while in practice it is common that only a subset of the followers has direct access to the leader. Although applicable to this case, the leader-following consensus laws derived in [9,10] rely on the neighbor's acceleration, which is difficult to obtain. In the case that the leader's angular velocity or acceleration has a linearly parameterized structure known to all followers, adaptive algorithms were constructed in [11,12] to recover the leader's motion. In order to obtain the reference trajectory with milder restrictions, finite-time convergent observers were proposed in [14–18] following the spirit of the sliding mode estimators of [22]. In addition, when the absolute and relative angular velocities are unavailable to each member, first-order filters of different forms were developed in [1,6,10,15] to generate the necessary damping. Such way of damping injection actually utilizes the passivity property of the system dynamics [19]. The studies in [16–18] attempted another means and designed various observers to provide direct velocity estimates. The resultant output-feedback consensus laws in [16–18], however, lead to semi-global stability and intrinsically require high gains to expand the domain of attraction. Internal model and adaptive control techniques were employed in [20] to cope with the inertia uncertainties and periodic disturbances for leader-following spacecraft. Recently, Nazari et al. [21] have developed a simultaneous position and attitude consensus algorithm for a leaderless spacecraft formation that can tolerate constant communication delays.

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https://doi.org/10.1016/j.ast.2017.12.015
1270-9638/

Note that all the preceding attitude consensus schemes, either asymptotically or exponentially stable, result in infinite settling time. In contrast, finite-time stability implies a finite convergence time and better disturbance rejection than asymptotic or exponential stability, as shown for the control of linear systems and Euler–Lagrange systems [22–24]. Given these advantages, finite-time attitude consensus laws were designed with full-state measurements [25–27] or attitude-only measurement [28–30]. The designs in [25] and [26] both utilized the adding-a-power-integrator approach but were based on the modified Rodrigues parameters and the vector part of unit quaternions respectively. Consequently, the controller in [25] does not allow rotations beyond 360° while the controller in [26] has a singularity at rotations of 180°. Note that the methods in [28,29] are limited to a stationary reference attitude. Generic time-varying reference trajectories were considered in [30] and feedback domination plus homogeneous techniques were applied to derive the velocity-free consensus law, which ensures merely semi-global finite-time stability. Finite-time consensus laws were also derived in [31] for relative planar motion control of a group of spacecraft.

Another important issue is that the attitude manifold admits no continuous feedback law with global attraction [32]. In addition, unit quaternions cover SO(3) twice and thus always induce two points representing the same desired attitude. Due to these features, most of the above attitude consensus laws are nonglobal and some can cause unnecessary full rotations even for small initial attitude errors, an undesirable phenomenon known as unwinding. To overcome this problem, discontinuous techniques were applied first in a memoryless manner [3] and then with hysteresis [33–35] for attitude synchronization. The resultant discontinuous control torque, however, cannot be implemented by actuators that provide continuous inputs only. The continuous synchronization laws in [2,5] avoid unwinding but only local stability was verified. In addition, the designs in [17,28–30] restrict attitude rotations to within 180°, which might not hold for large angle maneuvers. In fact, as shown in [36] for the attitude stabilization of a single rigid body, the best achievable result with continuous feedback is almost global finite-time stability (AGFTS). In other words, finite-time convergence generally holds except for a nowhere dense subset. However, it is not straightforward to extend the results for single rigid body to networked rigid bodies, as the information flow within the network must be taken into account.

This paper investigates the attitude consensus of networked rigid bodies, under either a leader-following or leaderless architecture, based on the unit quaternion parameterization. Distributed finite-time attitude consensus laws are derived, first with attitude plus angular velocity measurements and then with attitude-only measurements, via Lyapunov theory and homogeneous techniques. For the leader-following case, distributed sliding mode observers are designed to recover the leader's trajectory information in finite time. When angular velocity measurements are unavailable, quaternion filters are constructed to provide the necessary damping for the closed-loop system. This avoids the requirement of observers for direct estimation of angular velocity. The main contributions of this paper are summarized as follows:

1) The leader-following and leaderless attitude consensus issues are approached in a unified framework. The leaderless consensus laws can be readily obtained from the leader-following consensus laws by cutting the goal-seeking feedback and setting the inertial frame instead of the leader's frame as the reference attitude trajectory.

2) It is shown that the proposed methods attain finite-time attitude consensus for almost all initial conditions when the communication topology is connected and acyclic (i.e., a tree). Importantly, the distributed finite-time observer enables to derive attitude consensus laws by slightly extending the attitude controllers for single spacecraft. Although the observer for the leader's motion data and the controllers for attitude consensus are designed independently, a separation principle between them can be easily established. In other words, when the leader's information used in the controllers is replaced by the observers' estimates, the closed-loop trajectory never blows up in a finite time and thus finite-time attitude consensus is still achieved.

3) Since the double-covering feature of the unit quaternion representation is taken into account, our methods produce consistent continuous vector fields on the attitude manifold. As a result, the antipodal equilibria representing the same attitude are made both locally finite-time stable and thus the unwinding problem is avoided. In addition, the resulting control torques possess a simple proportional-derivative structure and are bounded *a priori*, thus facilitating the accommodation of saturation constraints.

The rest of this paper is organized as follows. In the next section, preliminary concepts, useful lemmas and equations of rigid-body attitude motion are introduced. Distributed attitude consensus laws for both leader-following and leaderless rigid-body networks are then developed in Section 3 with full state information and in Section 4 with attitude-only measurements, respectively. Section 5 demonstrates the application and effectiveness of the proposed methods via numerical simulations. Conclusions are summarized in Section 6.

2. Preliminaries and mathematical models

2.1. Notations

Throughout the paper, let $\|\cdot\|$ and $\|\cdot\|_\infty$ denote the Euclidean norm and infinity norm of a vector or a square matrix, respectively. Let $\mathbb{I}_n$ denote the index set $\{1,\cdots,n\}$. Set $\mathbf{1}_n=[1,\cdots,1]^T\in\mathbb{R}^n$ and denote by $\boldsymbol{I}_n$ the $n\times n$ identity matrix. For $\forall\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^3$, $\boldsymbol{x}^\times$ is the skew-symmetric matrix satisfying $\boldsymbol{x}^\times\boldsymbol{y}=\boldsymbol{x}\times\boldsymbol{y}$, where $\times$ is the cross product on $\mathbb{R}^3$. Denote by $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ the maximum and minimum singular values of a matrix. For any $x\in\mathbb{R}$, let $\mathrm{sgn}^\alpha(x)=\mathrm{sgn}(x)|x|^\alpha$, where $\alpha\geq 0$ and $\mathrm{sgn}(\cdot)$ is the standard sign function. Obviously, $\mathrm{sgn}^\alpha(x)$ is a continuous nonsmooth function when $0<\alpha<1$. For $\forall\boldsymbol{x}\in\mathbb{R}^n$ and $0\leq\alpha\leq 1$, define $\mathrm{sgn}^\alpha(\boldsymbol{x})=[\mathrm{sgn}^\alpha(x_1),\cdots,\mathrm{sgn}^\alpha(x_n)]^T$ and $\mathrm{sat}_\alpha(\boldsymbol{x})=[\mathrm{sat}_\alpha(x_1),\cdots,\mathrm{sat}_\alpha(x_n)]^T$, where $\mathrm{sat}_\alpha(x_i)=\mathrm{sgn}(x_i)\min\{|x_i|^\alpha,1\}$, $i\in\mathbb{I}_n$. In addition, $y=\mathcal{O}(x)$ means $|y|\leq c|x|$ for sufficiently small $|x|$ and some constant $c>0$. Given $\varepsilon>0$ and a weight vector $\boldsymbol{r}=(r_1,\cdots,r_n)$ with $r_i>0$, $i\in\mathbb{I}_n$, a dilation operator $\Delta_\varepsilon^{\boldsymbol{r}}$ is defined by $\Delta_\varepsilon^{\boldsymbol{r}}\boldsymbol{x}=(\varepsilon^{r_1}x_1,\cdots,\varepsilon^{r_n}x_n)$ for $\boldsymbol{x}\in\mathbb{R}^n$. For time-dependent functions and systems, the dilation operator $\Delta_\varepsilon^{\boldsymbol{r}}$ is extended as $\Delta_\varepsilon^{\boldsymbol{r}}(\boldsymbol{x},t)=(\Delta_\varepsilon^{\boldsymbol{r}}\boldsymbol{x},t)$.

2.2. Definitions and lemmas

Consider a time-varying system

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x},t),\quad \boldsymbol{x}\in\mathbb{R}^n \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x},t)=[f_1(\boldsymbol{x},t),\cdots,f_n(\boldsymbol{x},t)]^T\in\mathbb{R}^n$ is continuous with respect to $\boldsymbol{x}$. The vector field $\boldsymbol{f}(\boldsymbol{x},t)$ is said to be homogeneous of degree $h\in\mathbb{R}$ with respect to a dilation $\Delta_\varepsilon^{\boldsymbol{r}}$ if $f_i(\Delta_\varepsilon^{\boldsymbol{r}}(\boldsymbol{x},t))=\varepsilon^{r_i+h}f_i(\boldsymbol{x},t)$ for $\forall i\in\mathbb{I}_n$, $\forall\boldsymbol{x}\in\mathbb{R}^n$, and any $\varepsilon>0$ [37]. Denote by U a neighborhood of $\boldsymbol{x}=0$ and assume $\boldsymbol{f}(0,t)=0$. Then, $\boldsymbol{x}=0$ is uniformly finite-time stable if it is 1) uniformly Lyapunov stable in U and 2) uniformly finite-time convergent in U. If $U=\mathbb{R}^n$, then the origin is uniformly globally finite-time stable (UGFTS). If system (1) is time-invariant, the qualifier '*uniformly*' can be omitted in the preceding statements.

Lemma 2.1 *([38]). Consider the system*

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})+\hat{\boldsymbol{f}}(\boldsymbol{x},t),\quad \boldsymbol{f}(0)=0,\ \boldsymbol{x}\in\mathbb{R}^n \tag{2}$$