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# Distributed adaptive synchronization for multiple spacecraft formation flying around Lagrange point orbits

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ABSTRACT

This paper presents a distributed adaptive control framework for multiple spacecraft formation flying around Lagrange point orbits, which account for unmeasurable velocities and (spacecraft) mass uncertainties. The nominal trajectory for the formation system is a halo orbit parameterized by Fourier series expansions. Such an explicit, albeit approximate, description of the nominal trajectory facilitates each spacecraft in formation to include the relative state information into a cooperative feedback control system design, so that the relative motion can be driven towards a desired trajectory while maintaining a group synchronization during the maneuver. The developed distributed control strategies rely on the protocols formulated on an undirected topology with mutual information interactions, utilizing every available neighbor-to-neighbor communication data couplings, in order to improve the reliability of the formation. Numerical simulations show that the proposed adaptive control laws guarantee global asymptotic convergence and system robustness.



## 1. Introduction

Formation flying enables multiple vehicles to operate closely to accomplish complex space tasks that would be difficult to obtain with a single, conventional, spacecraft. Exploiting a task distribution among smaller, less-expensive vehicles, the spacecraft in a formation are able to share information and operate cooperatively, thus enhancing the system flexibility and reducing the overall mission costs [1]. Besides, formation flying also provides a means to improve specialized functions, such as image resolution and in-situ observation in astronomical missions [2]. Due to its distinctive peculiarity, some advanced formation flying-based mission scenarios have been proposed in the last decades in both geocentric and deep space environment. In this context, interesting examples are offered by PRISMA [3], a demonstration mission for autonomous technologies and on-orbit-servicing techniques, and Darwin [4], a scientific mission for Earth-like exo-planet detections.

One of the most practical applications of the formation flying concept is to observe (or explore) the celestial bodies by placing a number of spacecraft around the Lagrange points, known as the five equilibrium (stationary) solutions to the circular restricted three-body problem (CR3BP) [5]. A peculiarity of missions carried out near the natural (or artificial [6–8]) Lagrange points is that the formation may operate with an unobstructed view and is rarely affected by planetary perturbations (e.g. atmospheric and geomagnetic forces). For example, in the Sun–Earth system, the halo (or Lissajous) orbits in the vicinity of $L_2$ point naturally avoid the Sun eclipse, and are therefore suitable for measuring the cosmic microwave background. Also, orbits around $L_1$ point are never shadowed by Earth, and always view its sunlit hemisphere. Therefore, they usually serve as an interplanetary early warning storm monitor for solar disturbances [9].

Even though formation flying around Lagrange point orbits provides such valuable features as low-cost replacement of a faulty agent, it also poses a great challenge. In fact, since orbits around the collinear points are inherently unstable, a continuous active control is necessary to achieve long-term bounded relative motion. To that end, a number of formation control algorithms have been discussed, which can be roughly categorized into tight [10–13] and loose [14–16] strategies. The tight control method consists in stabilizing the spacecraft relative motion with respect to a specified nominal trajectory, using Lyapunov or eigenvalue stability theorem. The loose control concept, instead, relies on invariant manifolds theory, using the fact that the relative motion evolves and is always restricted within a bounded region provided some natural low-drift regions are found.

Thus far, much effort has been devoted to the study of formation flying around Lagrange point orbits, however several existing

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**Nomenclature**

| | | |
|---|---|---|
| $a_k^c, a_k^s, b_k^c, b_k^s, c_k^c, c_k^s$ | $k$-th order coefficients of Fourier series | |
| $\boldsymbol{e}$ | relative position errors | km |
| $\mathcal{E}$ | set of edges | |
| $\boldsymbol{f}$ | control force | N |
| $\mathbb{G}$ | adjacency matrix (with entries $[g_{ij}]$) | |
| $\mathcal{G}$ | communication topology graph | |
| $\mathbb{I}$ | identity matrix | |
| $\mathbb{L}$ | Laplacian matrix (with entries $[l_{ij}]$) | |
| $m$ | mass | kg |
| $N$ | number of spacecraft | |
| $n$ | order of Fourier series | |
| $n_r$ | angular velocity of relative orbit | rad/day |
| $\mathbb{O}$ | zero matrix | |
| $O$ | reference frame origin | |
| $\boldsymbol{r}$ | position vector (with $r = \lVert\boldsymbol{r}\rVert$), | au |
| $S$ | spacecraft | |
| $t$ | time | days |
| $T$ | period of nominal halo orbit | days |
| $\mathcal{T}$ | rotating reference frame | |
| $\boldsymbol{u}$ | propulsive (control) acceleration | m/s$^2$ |
| $\mathcal{V}$ | set of vertices | |
| $x, y, z$ | components of position vector in rotating frame | |
| $\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}$ | unit vectors of rotating coordinate axes | |
| $\boldsymbol{X}$ | state vector (with $\boldsymbol{X} \triangleq \left[\boldsymbol{r}^T, \dot{\boldsymbol{r}}^T\right]^T$) | |
| $\Delta v$ | velocity change | m/s |
| $\boldsymbol{\Phi}$ | state-transition matrix | |
| $\mu$ | normalized Earth mass | |
| $\rho_x, \rho_y, \rho_z$ | components of relative position vector | km |
| $\boldsymbol{\rho}$ | relative position vector with respect to nominal orbit | km |
| $\upsilon$ | vertex | |
| $\boldsymbol{\omega}$ | angular velocity vector of rotating frame (with $\omega = \lVert\boldsymbol{\omega}\rVert$) | rad/day |

*Subscripts*

| | |
|---|---|
| 0 | initial value |
| $ES$ | Earth-spacecraft |
| $f$ | final value |
| $h$ | halo orbit |
| $i$ | $i$-th spacecraft |
| $SS$ | Sun-spacecraft |

*Superscripts*

| | |
|---|---|
| T | transpose |
| $\star$ | desired value |
| $\cdot$ | time derivative |
| $\wedge$ | unit vector |
| $\sim$ | estimated value |

problems yet need to be solved. First, the non-integrability nature of the CR3BP prevents from any analytical representation of a nominal trajectory. Although some closed-form solutions have been discussed in the literature, however, they rely on a linearization procedure or on perturbation expansion-based approximations [5,17]. The linearized solution is well suited only for small-distances (relative to the Lagrange points), whereas the typical example of an algebraic solution is the well-known third-order approximation discussed by Richardson [17], which however leads to a remarkable deviation from the nominal orbit after about one half period only. In addition, previous studies [18,19] on formation keeping algorithms around Lagrange point orbits assume the neighboring spacecraft velocity and its mass to be precisely known, which is usually a demanding task for an onboard measuring system. To reduce the operating costs and the spacecraft weight, the problem of guaranteeing the system stability even in the presence of velocity or mass uncertainties becomes crucial, especially when faults or high noises reduce the effectiveness of the on-board sensors. Finally, most of the existing works [10–13,19,20] on this issue are limited to a leader-follower formation structure, rendering an inherent weakness that the leader is a single point of failure for the whole system. To mitigate these risks, the system robustness and its overall redundancy need to be strengthened.

Recognizing these open issues, a distributed framework of multiple spacecraft formation flying around Lagrange point orbits is here discussed. The contributions of this paper are twofold. First, the nominal trajectory is parameterized via high-order Fourier series expansions. Unlike the classical third-order solution that suffers from a huge amount of algebraic manipulations, the Fourier series-based solution relies on a least-square approach and provides a better accuracy as the order of the expansion series increases. Note that the Fourier series-based approach has been recently [21] used to continuate the spacecraft orbit, for long term, in the real Solar System model. Compared to the existing works, the approximate closed-form description of the nominal trajectory captures most nonlinearity. This results in a propellant reduction necessary to maintain the formation around the nominal trajectory. Second, two distributed adaptive synchronization control strategies are proposed to account for unmeasurable spacecraft velocities and mass uncertainties. By exploiting the available information exchange among the formation, every spacecraft updates its state using the data flow transmitted from its local neighbours (not necessarily limited to the nearest one, as is discussed in Ref. [22]), so that the overall redundancy and group robustness are enhanced. Besides, the proposed consensus-based control law also guarantees a time-balanced (synchronization), as well as a high tracking accuracy.

This paper is organized as follows. Section 2 illustrates the mathematical model in the Sun–Earth CR3BP, and presents an approximate analytical solution to the nominal orbit via Fourier series expansions. Section 3 provides two distributed control strategies using mutual information couplings to account for unmeasurable velocities and mass uncertainties, respectively. The control effectiveness is then investigated in Section 4 by means of some numerical simulations. Finally, some concluding remarks are given in Section 5.

## 2. Problem formulation

In this section, the Sun and Earth are considered as the two primary bodies, and the halo orbit around $L_2$ point is designated as the nominal trajectory. To describe the relative motion equation of the formation system, it is useful to introduce first the mathematical model used in CR3BP.

### 2.1. Equations of motion

The dynamic model describing the Sun–Earth CR3BP is formulated in a classical rotating coordinate system $\mathcal{T}\left(O; \hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\right)$, see Fig. 1. The origin $O$ of the reference frame is centered at the system barycenter, while the plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}})$ coincides with the ecliptic plane, $\hat{\boldsymbol{x}}$ axis points to the Earth and $\hat{\boldsymbol{z}}$ axis is positive in the direction of the angular velocity vector $\boldsymbol{\omega}$. For convenience, a normalized set of units is introduced, such that the total mass of